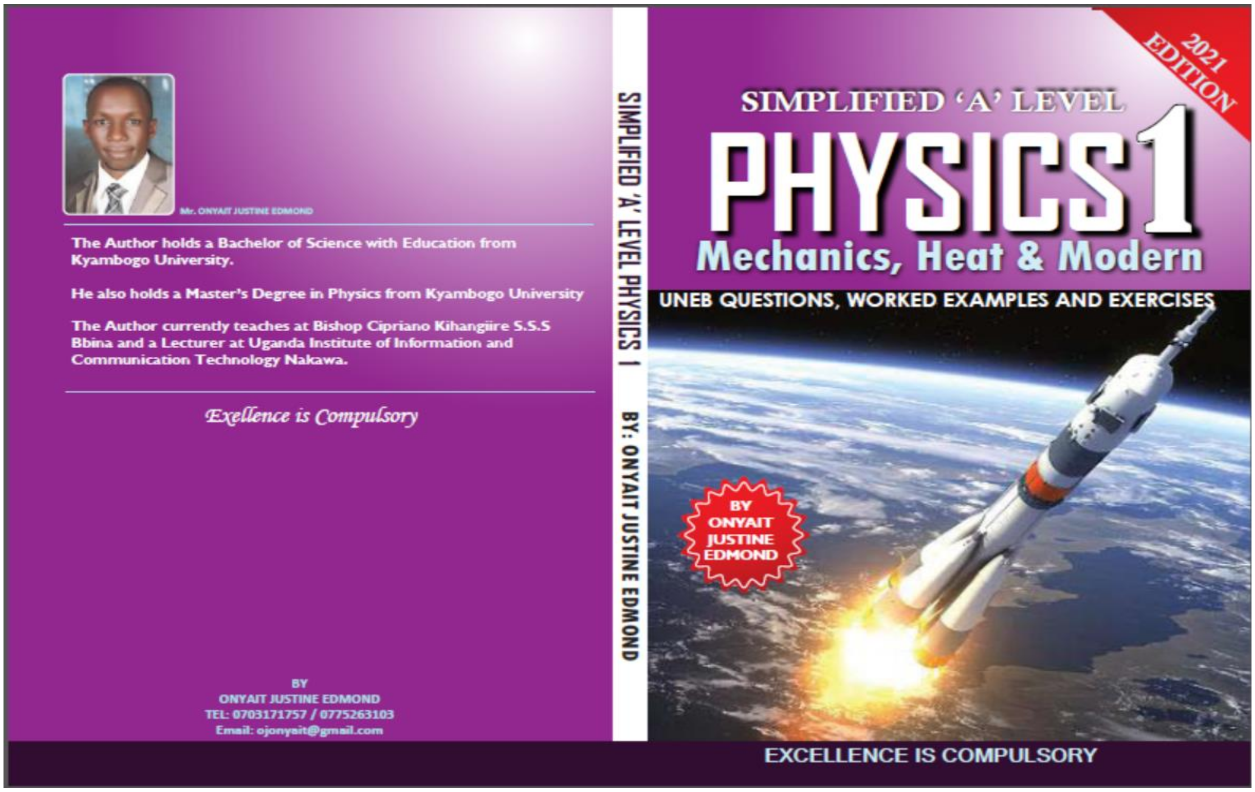


PREFACE

This book covers Mechanics, Heat and Moder physics. The content is brief, concise and summarised with marking points underlined in most cases.

A variety of examples have been presented in the book to help a student easily understand numerical calcultions in physics 1 even with out the help of a teacher. A number of exercises have also been included in every chapter and at the end of each numerical calculation question, the answer is placed in high lighted brackets.

At the end of every chapter, you will find UNEB questions and their answers dating from 1998 to 2020.

This book has been embraced and used but not limited to schools such as Seeta high school, St Marys College Kitende, Kings college Budo, Bishop Cipriano Kihangire s.s.s.

For Further assistance, do not hesitate to consult the author on watapp number 0775263103 or direct call on the same number or 0703171757.

Students who need online or face to face tutorials can also reach the author through the above contacts.

SECTIONA: MECHANICS

CHAPTER1: DIMENSIONS OF A PHYSICAL QUANTITY

1.1.0: Fundamental quantities

These are quantities which can't be expressed in terms of any other quantities by using any mathematical equation. E.g.

Mass - M Length - L Time- T

1.1.1: Derived quantities

These are quantities which can be expressed in terms of the fundamental quantities of mass, length, and time e.g.

i) Pressure
ii) Acceleration
iii) Momentum
iv) Density

1.1.2: DIMENSIONS OF A PHYSICAL QUANTITY

This refers to the way a physical quantity is related to the three fundamental quantities of length, mass and time.

Or It refers to the power to which fundamental quantities are raised.

Symbol of dimensions is [ ]

Examples

[Area] $= L^2$, [Volume]$= L^3$

$[Density] = \frac{[Mass]}{[Volume]} = \frac{M}{L^3}$ =ML^{-3}

[Velocity]$= \frac{[Displacement]}{[Time]} = \frac{L}{T} = LT^{-1}$

[Acceleration]$= \frac{[Change\ in\ Velocity]}{[Time]} = \frac{LT^{-1}}{T} = LT^{-2}$

[Momentum]=[Mass][Velocity]=MLT^{-1}

[Weight]=[m][g]=MLT^{-2}

[Force]=[Mass][Acceleration]= MLT^{-2}

[Pressure]$= \frac{[Force]}{[Area]} = \frac{MLT^{-2}}{L^2}$ =M$L^{-1}T^{-2}$

1.1.3: USES OF DIMENSIONS

1. Used to check the validity of the equation or check whether the equation is dimensionally consistent or correct.
2. Used to derive equations

a) Checking validity of equations (dimensional consistency)

When the dimensions on the L-H-S of the equations are equal to the dimensions on the R-H-S, then the equation is said to be dimensionally consistent.

Examples

1. The period T, of a simple pendulum is given by $T = 2\pi\sqrt{\frac{l}{g}}$ Show that the equation is dimensionally correct.

Where $2\,\pi =$ dimension less constant $g =$ Acceleration due to gravity

$l =$ length of pendulum

Solution

L.H.S $[T] = T$

R.HS $= \left[2\pi\sqrt{\frac{l}{g}}\ \right] = \left[2\pi\left(\frac{l}{g}\right)^{\frac{1}{2}}\right] = [2\,\pi]\left(\frac{[l]}{[g]}\right)^{\frac{1}{2}}$

$= \left(\frac{L}{LT^{-2}}\right)^{\frac{1}{2}} = (T^2)^{1/2}$ = T

Since the dimensions on the L.H.S are equal to the dimensions on the R.H.S then the equation is dimensionally consistent.

Example

Show that the equation $v^2 = u^2 + 2as$ is dimensionally correct.

Solution

L.H.S $[v^2] = (LT^{-1})^2 = L^2T^{-2}$

R.H.S $= [u^2] =$ [2as]$= (LT^{-1})^2 = 1x\ LT^{-2}L$

$= L^2T^{-2} = L^2T^{-2}$

Since dimensions on the L.H.S are equal to dimensions on the R.H.S then the equation is dimensionally correct.

Exercise: 1

1. Show that the following equations are dimensionally consistent where symbols have their usual meanings

   i) $S = ut + \frac{1}{2}at^2$ ii) $v = ut + at$ iii) $Ft = mv - mu$

2. The frequency f of vibration of the drop of a liquid depends on surface tension, γ of the drop, its density, ρ and radius r of the drop. Show that $f = k\sqrt{\frac{\gamma}{\rho r^3}}$ where k is a non-dimensional constant

UNEB 2016 NO 1 (a)

(i) Define dimensions of a physical quantity. (01mark)

(ii) In the gas equation

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

Where P= pressure, V= volume, T=absolute temperature, and R= gas constant. What are the dimensions of the constants a and b. (04marks)

UNEB 2010 NO 4 (d)

The velocity V of a wave in a material of young modulus E and density ρ is given by $V = \sqrt{\left(\frac{E}{\rho}\right)}$

Shows that the relationship is dimensionally correct (03 marks)

UNEB2009 No 3b

A cylindrical vessel of cross sectional area, A contains air of volume V, at pressure p trapped by frictionless air tight piston of mass, M. The piston is pushed down and released.

i) If the piston oscillates with simple harmonic motion, shows that its frequency f is given

$f = \frac{A}{2\pi}\sqrt{\frac{\mathrm{p}}{\mathrm{MV}}}$ (06 marks)

ii) Show that the expression for f in b(i) is dimensionally correct (03 marks)

UNEB 2001 No 2 b

The velocity V of sound travelling along a rod made of a material of young's modulus y and density ρ is given by $V = \sqrt{\frac{Y}{\rho}}$ Show that the formula is dimensionally consistent (03 mks)

UNEB 1997 No 1

a) i) What is meant by dimensions of a physical quantity (1mk)

ii)The centripetal force required to keep a body of mass m moving it a circular path of radius r is given by $F = \frac{m\,v^2}{r}$ show that the formula is dimensionally consistent. (04 marks)

CHAPTER 2: COMPOSITION AND RESOLUTION OF VECTORS

VECTOR QUANTITY

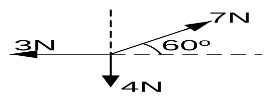
It is a physical quantity with both magnitude and direction.
Example; displacement, velocity, acceleration, force, weight and momentum

SCALAR QUANTITY

It is a physical quantity with only magnitude.
Example; distance, speed, time, temperature, mass and energy

Example

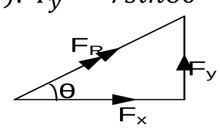
A particle at the origin O is acted upon by the three forces as shown below. Find the position of the particle after 2 seconds of its mass is 1kg.



Solution

$(\rightarrow)$: $F_x = -3 + 7cos60 = 0.5N$

$(\uparrow)$: $F_y = 7sin60 - 4 = 2.06N$



$$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{0.5^2 + 2.06^2} = 2.12N$$

But $F_R = ma$

$2.12 = 1a$

$a = 2.12ms^{-2}$

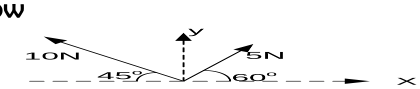
From $S = ut + ½\, at^2$

$u = 0 \quad t = 2s \quad a = 2.12ms^{-2}$

$S = 0x2 + ½\, x2.12x2^2 = 4.24m$

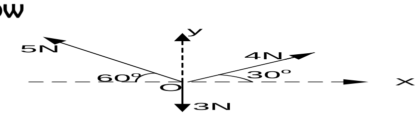
EXERCISE:2

1. Two coplanar forces act on a point O as shown below



Calculate the resultant force **An[12.3N at 68.0° above the horizontal]**

2. Three coplanar forces act at a point as shown below



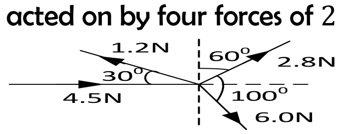
Find the resultant force acting at O **An[3.4N at 73.1° above the horizontal]**

UNEB 2019 No1

(a) (i) Distinguish between scalar quantity and vector quantity. (01 mark)

(ii) Give two examples of each type of quantity. (02 marks)

(b) A body of mass 0.2 kg at rest is acted on by four forces of $2.8N, 6.0N, 4.5N$ and $1.2N$ as shown below.



Calculate the:

(i) Resultant force on the body **An[8.73N at** $24.6°$**]** (04 marks)

(ii) Distance moved in 4s. **An[349.2m]** (02 marks)

CHAPTER 3: KINEMATICS

Kinematics is the branch of physics which deals with motion of bodies and systems without consideration of the force causing motion

(a) LINEAR MOTION

Acceleration

It is the rate of change of velocity with time

It SI unit is ms^{-2}

Acceleration = $\frac{change\ in\ velocity}{time}$

$$a = \frac{v-u}{t}$$

Uniform acceleration

Constant rate of change of velocity.

Equations of uniform acceleration

1st equation

Suppose a body moving in a straight line with uniform acceleration a, increases its velocity from u to v in a time t, then from definition of acceleration

$$a = \frac{v-u}{t}$$

$$at = v - u$$

$\boxed{v = u + a\,t}$................1

2nd equation

Suppose an object with velocity u moves with uniform acceleration for a time t and attains a velocity v, the distance s travelled by the object is given by S = average velocity x time

$S = \left(\frac{v+u}{2}\right)t$ But $v = u + at$

$$S = \frac{(u + at + u)}{2}t$$

$$S = \frac{(2u + at)t}{2}$$

$$S = \frac{2ut + at^2}{2}$$

$\boxed{S = ut + \frac{1}{2}\,at^2}$........................2

3rd equation

S = average velocity x time

$S = \left(\frac{v+u}{2}\right)t$ But $t = \frac{v-u}{a}$

$$S = \left(\frac{v+u}{2}\right)\left(\frac{v-u}{a}\right)$$

S = $\frac{v^2-u^2}{2a}$

$\boxed{v^2 = u^2 + 2as}$...................3

Examples

1) A particle moving in a straight line with a constant acceleration of $2ms^{-2}$ is initially at rest, find the distance covered by the particle in the 3rd second of its motion.

Solution

Using S = $ut + \frac{1}{2}\,at^2$

u=0m/s, t=2s and t=3s a= $2ms^{-2}$

t=2 : $s = 0x2 +$ ½ $x2x2^2 = 4m$

When t=3: a=$2ms^{-2}$ u=0m/s

$s = 0x3 +$ ½ $x2x3^2 = 9m$

Distance in 3rd

Distance for 3s – distance for 2s

$= 9 - 4 = 5m$

Distance in 3rds in 5m

2) A Travelling car A at a constant velocity of 25m/s overtake a stationery car B. 2s later car B sets off in pursuit , accelerating at a uniform rate of $6ms^{-2}$. How far does B travel before catching up with A

Solution

For A: $S_A = ut + \frac{1}{2}\,at^2$

Since it moves with a constant velocity a=0

S_A = 25t---------------(1)

For B: $S_B = ut + \frac{1}{2}at^2$

If B is to catch up with A then it must travel faster i.e it will take a time of (t-2)s

$S_B = 0x(t-2) + ½ x6(t-2)^2$

$S_B = 3t^2-12t+12$..............(2)

For B to catch A then

$S_A = S_B$

$25t = 3t^2 - 12t + 12$

$3t^2 - 37t + 12 = 0$

$t = \frac{37 \pm \sqrt{37^2 - 4x12x3}}{2x3}$

$t=12s$ or $t = \frac{1}{3}s$

Since the car leaves 2s later then time 12s is correct since it gives a positive value

$S_B = 25x12$

$S_B = 300m$

EXERCISE:3

1. A particle which is moving in a straight line with a velocity of $15ms^{-1}$ accelerates uniformly for 3.0s, increasing its velocity to $45ms^{-1}$. What distance does it travel while accelerating? **An[90m]**
2. A bus travelling steadily at 30m/s along a straight road passes a stationary crab which, 5s later, begins to move with a uniform acceleration of $2ms^{-2}$ in the same direction as the bus
   (a) How long does it take the car to acquire the same speed as the bus
   (b) How far has the car travelled when it is level with the bus **An[15s, 1181m]**
3. A body accelerates uniformly from rest at the rate of $6ms^{-2}$ for 15 seconds. Calculate
   i) velocity reached within 15 seconds
   ii) the distance covered within 15 seconds
   An[90m/s, 675m]
4. An electron in a TV tube reaches a velocity in the region of $10^7 ms^{-1}$. If the distance between the filament and the accelerating anode is 5cm, what is the acceleration of the electron? **An[$10^{15}ms^{-2}$]**

(b)

VERTICAL MOTION UNDER GRAVITY

Definition

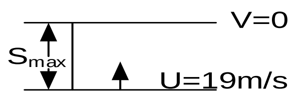
Acceleration due to gravity (g) is rate of change of velocity with time for an object falling freely under gravity.

OR The force of attraction due to gravity exerted on a 1kg mass.

Numerical examples

1. A particle is projected vertically upwards with velocity of $19.6ms^{-1}$. Find
   i) The greatest height attained
   ii) Time taken by the particle to reach maximum height
   iii) Time of flight

Solution



At greatest height $v = 0m/s$

$v^2 = u^2 - 2gs$

$0^2 = 19.6^2 - 2x9.81\ s_{max}$

$s_{max} = \frac{19.6^2}{2x9.81} = 19.58m$

ii) From $v = u - gt$

$u = 19.6, g = 9.81ms^{-2}$ $v = 0$ at max height

$0 = 19.6 - 9.81t$

$t = 1.998s$

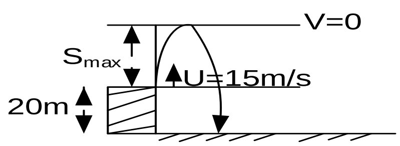
Time to maximum height $= 2.0s$

iii) Time of flight $= 2x$ time to max height

$= 2x2 = 4.0s$

2. A man stands on the edge of a cliff and throws a stone vertically upwards at $15ms^{-1}$. After what time will the stone hit the ground 20m below the point of projection

Solution



v=0m/s at max height, s_{max}=? t=?

Method I: $v = u - gt$

$0 = 15 - 9.81t$

$t = 1.53s$

Time to maximum height $= 1.53s$

$v^2 = u^2 + 2gs$

$0 = 15^2 - 2x9.81s_{max}$

$$s_{max} = \frac{15^2}{2x9.81} = 11.47m$$

Maximum height $= 11.47m$

Total height $= (11.47 + 20) = 31.47m$

When the ball begins to return down from max height $u = 0m/s$

$$S = ut + \frac{1}{2}gt^2$$

$31.47 = 0xt + ½ x9.81t^2$

$$t = \sqrt{\frac{31.47x2}{9.81}} = 2.53s$$

Total time $= (2.53 + 1.53) = 4.06s$

Time taken to hit the ground $= 4.06s$

Method II

The height of the cliff = 20m which is below the point of project therefore

$s = -2m\ u = 15m/s$

$$S = ut - \frac{1}{2}gt^2$$

$-20 = 15t - ½ x9.81t^2$

$-20 = 15t - 4.905t^2$

$t = 4.06s$

Time taken to hits the ground $= 4.06s$

Exercise :4

1. A pebble is dropped from rest at the top of a cliff 125m high.
   (a) How long does it take to reach the foot of the cliff and with what speed does it hit the floor
   (b) With what speed must a second pebble be thrown vertically down wards from the cliff top if it is to reach the bottom in 4s .
   An(5s, 50m/s, 11.25m/s)
2. A body dropped from rest falls half its total path in the last second before it strikes the ground. Form what height was it dropped **An**[$58.2m$]

UNEB 2014 No 1(c)

(i) State **Newton's laws of motion** (03marks)

(ii) Explain how a rocket is kept in motion (04marks)

(iii) Explain why passengers in a bus are thrown backwards when the bus suddenly starts moving. (03marks)

UNEB 2013 No 3(d)

(i) Define uniformly accelerated motion (03marks)

(ii) A train starts from rest at station **A** and accelerates at 1.25 m s^{-2} until it reaches a speed of 20 m s^{-1}. It then travels at this steady speed for a distance of 1.56km and then decelerates at 2 m s^{-2} to come to rest at station **B**. Find the distance from **A** and **B**

An (1 820m) (04marks)

UNEB 1993 No 1

(a) Define the terms
   (i) Displacement
   (ii) Uniform acceleration

(b) i) A stone thrown vertically upwards from the top of a building with an initial velocity of 10m/s. the stone takes 2.5s to land on the ground.
   ii) Calculate the height of the building
   (iii) State the energy changes that occurred during the motion of the stone (03 marks)

(c) PROJECTILE MOTION

This is the motion of a body which after being given an initial velocity moves freely under the influence of gravity

TERMS USED IN PROJECTILES

1. TIME OF FLIGHT [T]

It refers to the total time taken by the projectile to move from the point of projection to the point where it lands on the horizontal plane through the point of projection.

Vertically: $S_y = u_y$ t + ½ at²

at point A when the projectile return to the plane S_y=0,

t=T(time of flight), $a = -g \quad u_y = u \sin\theta$

$$0 = u\sin\theta T - \frac{gT^2}{2}$$

$$T\left(u\sin\theta - \frac{gT}{2}\right) = 0$$

Either $T = 0$ or $\left(u\sin\theta - \frac{gT}{2}\right) = 0$

$$\left(u\sin\theta - \frac{gT}{2}\right) = 0$$

$$u\sin\theta = \frac{gT}{2}$$

$$\boxed{T = \frac{2u\sin\theta}{g}}$$

Note: The time of flight is twice the time to maximum height

2. EQUATION OF A TRAJECTORY

A trajectory is a path described by a projectile.

A trajectory is expressed in terms of horizontal distance x and vertical distance y.

For horizontal motion at any time t

$$x = u\cos\theta t$$

$t = \frac{x}{u\cos\theta}$ ----------------[1]

For vertical motion at any time t

$y = u\sin\theta t - ½ gt^2$ -------------[2]

Putting t into equation [2]

$$y = u\sin\theta \frac{x}{u\cos\theta} - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

$$\boxed{y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}}$$

since $y = ax - bx^2$

the motion is parabolic

Either $\boxed{y = x\tan\theta - \frac{g\,x^2 Sec^2\theta}{2\,u^2}}$

Or $\boxed{y = x\tan\theta - \frac{g\,x^2(1+\tan^2\theta)}{2\,u^2}}$

Examples

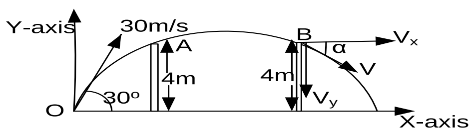
1. A Particle is projected with a velocity of 30ms⁻¹ at an angle of elevation of 30°. Find
   i) The greatest height reached
   ii) The time of flight
   iii) Horizontal range
   iv) The velocity and direction of motion at a height of 4m on its way upwards

Solution

(i) $Smax = \frac{u^2\sin^2\theta}{2g} = \frac{30^2\sin^2 30}{2x9.81} = 11.47m$

(ii) $\boldsymbol{T} = \frac{\boldsymbol{2u\sin\theta}}{\boldsymbol{g}} = \frac{2\,x\,30\,sin30}{9.81} = 3.06s$

(iii) R= $\frac{u^2\sin2\theta}{g} = \frac{30^2\,sin2x30}{9.81} = 79.45m$



For vertical motion

$$y = u\sin\theta t - ½ gt^2$$

$$4 = 30\sin30t - ½\,9.81x\,t^2$$

$$4.905t^2 - 15t + 4 = 0$$

$t = 2.76s$ or $t = 0.30s$

The value of $t = 0.30s$ is the correct time since it's the smaller value for which the body moves upwards.

$$v_x = u\cos\theta$$

$$v_x = 30\cos 30 = 25.98m/s$$

$$v_y = u\sin\theta - gt$$

$$v_y = 305x30 - 9.81x0.30 = 12.06m/s$$

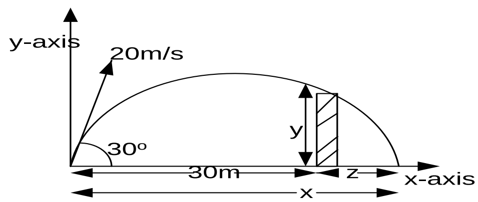
$v = \sqrt{V_x^2 + V_y^2} = \sqrt{25.98^2 + 12.06^2} = 28.64m/s$

Direction : $\alpha = tan^{-1}\frac{V_y}{V_x} = tan^{-1}\left(\frac{12.06}{25.98}\right)$ =**24.9°**

Velocity is 28.64m/s at 24.9° to horizontal

2. A ball is kicked from the spot 30m from the goal post with a velocity of 20m/s at 30° to the horizontal. The ball just clears the horizontal bar of a goal post. Find;
   (i) Height of the goal post
   (ii) How far behind the goal post does the ball land

Solution



horizontal motion : $x = ucos\theta xt$

$30 = 20cos30xt$

$t = 1.732s$

For vertical motion: $y = usin\theta t - ½\, gt^2$

$y = (20sin30)x1.732 - \frac{1}{2}x9.81x(1.732)^2$

$y = 2.61m$

Height of the goal post = 2.61m

ii) Time of flight

$T = \frac{2\text{ u sin}\theta}{\text{g}} = \frac{2x20xsin30}{9.81} = 2.04s$

iii) Horizontal distance: $x = ucos\theta xt$

$x = 20cos30x2.04$ =**35.33m**

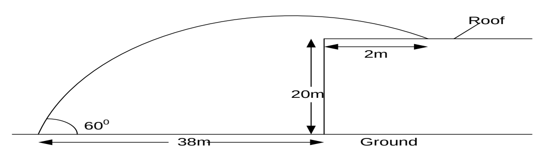
but $x = 20 + z$

$35.33 = 30 + z$

$z = 5.33m$ The ball 5.33m behind the goal

EXERCISE :5

1. A hammer thrown in athletics consists of a metal sphere of mass 7.26kg with a wire handle attached, the mass of which can be neglected. In a certain attempt it is thrown with an initial velocity which makes an angle of 45° with the horizontal and its flight takes 4.00s. stating any assumptions find;
   (i) The horizontal distance travelled
   (ii) Kinetic energy of the sphere just before it strikes the ground **An [80.0m, 2.90x10³J]**
2. A soft ball is thrown at an angle of 60 above the horizontal. It lands a distance 2m from the edge of a flat roof of height 20m. the edge of the roof is 38m horizontally from the thrower.



Calculate
   (i) The speed at which the ball was thrown **An (25.4 m**s^{-1})
   (ii) The velocity with which the ball strikes the roof . **An (15.64 m**s^{-1} $at\ 36.2°$ below the horizonta**l**)

UNEB 2016 No1 (b)

A particle is projected from a point on a horizontal plane with a velocity, u, at an angle, θ, above the horizontal. Show that the maximum horizontal range R_{max} is given by $R_{max} = \frac{u^2}{g}$ where g is acceleration due to gravity. (04marks)

UNEB 2014 No1 (a)

(i) What is a **projectile motion** (01marks)

(ii) A bomb is dropped from an aero plane when it is directly above a target at a height of 1402.5m. The aero plane is moving horizontally with a speed of 500kmh^{-1}. Determine whether the bomb will hit the target. **An (misses target by 2347.2m)** (05marks)

UNEB 2012 No 3 (d)

(i) Derive an expression for maximum horizontal distance travelled by a projectile in terms of the initial speed u and the angle of projection θ to the horizontal [02 marks]

(ii) Sketch a graph to show the relationship between kinetic energy and height above the ground in a projectile.

CHAPTER 4: NEWTON'S LAWS OF MOTION

LAW I : Everybody continues in its state of rest or uniform motion in **a straight line** unless acted upon by an external force.

This is sometimes called the law of **inertia**

Definition

Inertia is the reluctance of a body to start moving once it's at rest or to stop moving if its already in motion.

Explain why a passenger jerks forward when a fast moving car is suddenly stopped.

Passengers jerk forward because of inertia. When the car is suddenly stopped, the passenger tends to continue in uniform motion in a straight line because the force that acts on the car does not act on the passenger

LAW II: The rate of change of momentum of a body is directly proportional to the applied force and takes place in the direction of the force.

Consider a mass m moving with velocity u. If the mass is acted on by a force F and its velocity changes to v;

By Newton's law of motion

$F \ \alpha \ \frac{mv-mu}{t} = \frac{k(mv-mu)}{t} = km\frac{(v-u)}{t} = kma$

Since $a = \frac{v-u}{t}$

When $F = 1N$, $m = 1kg$ and $a = 1ms^{-2}$

$1 = kx1x1$

$k = 1$

$$\boxed{F = ma}$$

Note: F must be the resultant force

LAW III: To every action there is an equal but opposite reactions.

$$\boldsymbol{F_1 = -F_2}$$

Example of 3rd law of motion

- A gun moves backwards on firing it.
- A ball bounces on hitting the ground.

Rocket engine propulsion

Fuel is burnt in the combustion chamber and exhaust gases are expelled at a high velocity. This leads to a large backward momentum. From conservation of momentum an equal forward momentum is gained by the rocket, due to continuous combustion of fuel there is a change in the forward momentum which leads to the thrust hence maintaining the motion of the rocket

4.1.2: LINEAR MOMENTUM AND IMPULSE

Linear momentum (p) is the product of the mass and the velocity of the body moving in a straight line.

IMPULSE

This is the product of the force and time for which the force acts on a body

i.e. Impulse (I) = Force(F) x time (t)

$$\vec{I} = \vec{F}\, t$$

The unit of impulse is Ns.

An impulse produces a change in momentum of a body. If a body of mass(m) has it velocity changed from u to v by a force F acting on it in time t, then from Newton's 2^{nd} law.

$Ft = mv - mu$

$I = Ft$

$I = mv - mu$

Impulse = change in momentum

Examples

1. A body of mass 5kg is initially moving with a constant velocity of $2ms^{-1}$, when it experiences a force of 10N is 2s, find
   (i) The impulse given to the body by the force
   (ii) The velocity of the body when the force stops acting

Solution

$$I = ft = 10x2 = 20Ns$$
$$I = mv - mu$$
$$20 = 5v - 5x2$$
$$v = 6m/s$$

2. A girl of mass 50kg jumps onto the ground from a height of 2m. Calculate the force which acts on her when she lands
   (i) As she bends her knees and stops within 0.2 s
   (ii) As she keeps her legs straight and stops in 0.05s

Solution

i) $v^2 = u^2 + 2gs$

$v^2 = 0^2 + 2x9.81x2$

$v = \sqrt{39.24} = 6.03ms^{-1}$

Using $F = \frac{mv-mu}{t}$

$F = = \frac{50(6.03\text{-}0)}{0.2} = 1507.5N$

ii) $F = \frac{mv-mu}{t}$

$F = \frac{50(6.03-0)}{0.05} = 6030N$

4.1.3: WHY LONG JUMPER BEND KNEES

By bending the knees, the time taken to come to rest is increased, which reduces the rate of change of momentum, therefore the force on the jumpers legs is reduced thus less pain on the legs.

4.1.4: LAW OF CONSERVATION OF LINEAR MOMENTUM

It states that for a system of colliding bodies, their total linear momentum remains constant in a given direction provided no external forces acts on them.

Suppose a body A of mass m, and velocity U_1, collides with another body B of mass m_2 and velocity U_2 moving in the same direction

By principle of conservation of momentum

$$\underbrace{m_1u_1 + m_2u_2}_{\text{Total momentum before collision}} = \underbrace{m_1v_1 + m_2v_2}_{\text{Total momentum after collision}}$$

4.1.5: Proof of the law of conservation of momentum using Newton's law

Let two bodies A and B with masses m_1 and m_2 moving with initial velocities u_1 and u_2 and let their velocities after collision be v_1 and v_2 respectively for time t with ($v_1<v_2$)

By Newton's 2^{nd} law:

Force on m_1: $F_1 = \frac{m_1(v_1-u_1)}{t}$

Force on m_2: $F_2 = = \frac{m_2(v_2-u_2)}{t}$

By Newton's 3rd law: $\boldsymbol{F_1 = -F_2}$

$$\frac{m_1(v_1-u_1)}{t} = -\frac{m_2(v_2-u_2)}{t}$$
$$m_1v_1 - m_1u_1 = -m_2v_2 + m_2u_2$$
$$\therefore m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

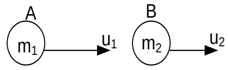
Hence $m_1u_1 + m_2u_2 =$ **constant**

Examples

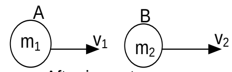
1. A particle P of mass m_1, travelling with a speed u_1 makes a head-on collision with a stationary particle Q of mass m_2. If the collision is elastic and the speeds of P and Q after impact are v_1 and v_2 respectively. Show that for $\beta = \frac{m_1}{m_2}$

   (i) $\frac{u_1}{v_1} = \frac{\beta+1}{\beta-1}$ (ii) $\frac{v_2}{v_1} = \frac{2\beta}{\beta-1}$

Solution



Before impact



After impact

By law of conservation of momentum

$m_1u_1 = m_1v_1 + m_2v_2$--------------[x]

$(u_1 - v_1) = \frac{m_2}{m_1}v_2$

Therefore $u_1 - v_1 = \frac{v_2}{\beta}$

$\beta(u_1 - v_1) = v_2$----------------------- [1]

for elastic collision k.e is conserved

$\frac{1}{2} m_1u_1^2 = \frac{1}{2} m_1{v_1}^2 + \frac{1}{2} m_2{v_2}^2$

$m_1(u_1^2 - v_1^2) = m_2({v_2}^2\ )$

$\frac{m_1}{m_2}({u_1}^2\ - {v_1}^2) = {v_2}^2$

$\beta(u_1^2 - v_1^2) = v_2^2$----------------------[2]

equating [1] and [2]

$\beta(u_1^2 - v_1^2) = [\beta(u_1 - v_1)]^2$

$\beta(u_1^2 - v_1^2) = \beta^2(u_1 - v_1)(u_1 - v_1)$

$(u_1 - v_1)\ (u_1 + v_1) = \beta(u_1 - v_1)(u_1 - v_1)$

$(u_1 + v_1) = \beta(u_1 - v_1)$

$v_1 + \beta v_1 = \beta u_1 - u_1$

$v_1(1 + \beta) = u_1(\beta - 1)$

$\frac{u_1}{v_1} = \frac{\beta+1}{\beta-1}$

ii) From $\frac{u_1}{v_1} = \frac{\beta+1}{\beta-1}$ ------------------ [xx]

from equation[1] : $v_2 = \beta(u_1 - v_1)$

$v_2 = \beta u_1 - \beta v_1$

$u_1 = \frac{v_2+\beta v_1}{\beta}$ put into (xx)

$$\frac{\left(\frac{v_2 + \beta v_1}{\beta}\right)}{v_1} = \frac{(1+\beta)}{(\beta - 1)}$$

$(v_2 + \beta v_1)(\beta - 1) = (1 + \beta)\beta v_1$

$\beta v_2 + \beta^2 v_1 - v_2 - \beta v_1 = \beta v_1 + \beta^2 v_1$

$\beta v_2 - v_2 = 2\beta v_1$

$v_2(\beta - 1) = 2\beta v_1$

$\frac{v_2}{v_1} = \frac{2\beta}{\beta-1}$

1. Ball P, Q and R of masses m_1, m_2 and m_3 lie on a smooth horizontal surface in a straight line. The balls are initially at rest. Ball P is projected with a velocity u_1 towards Q and makes an elastic collision with Q. if Q makes a perfectly in elastic collision with R, show that R moves with a velocity.

$$v_2 = \frac{2\, m_1m_2u_1}{(m_1 + m_2)(m_2 + m_3)}$$

Solution

Elastic collision of P and Q:

Conservation of momentum:

$m_1u_1 = m_1v_P + m_2v_Q$

$v_P = u_1 - \frac{m_2v_Q}{m_1}$......................................(1)

Conservation of kinetic energy:

$\frac{1}{2}m_1u_1^2 = \frac{1}{2}m_1v_P^2 + \frac{1}{2}m_2v_Q^2$......................(2)

Putting [1] into [2]

$m_1u_1^2 = m_1\left(u_1 - \frac{m_2v_Q}{m_1}\right)^2 + m_2v_Q^2$

$v_Q = \frac{2\, m_1u_1}{m_1+m_2}$......................................(3)

In elastic collision of Q and R:

$m_2v_Q + m_3 0 = (m_2 + m_3)v_2$

$m_2\frac{2\, m_1u_1}{m_1 + m_2} = (m_2 + m_3)v_2$

$v_2 = \frac{2\, m_1m_2u_1}{(m_1 + m_2)(m_2 + m_3)}$

2. A particle **P** of mass m_1 moving at a speed u_1 collides head on with a stationery particle Q of mass m_2. the collision is perfectly elastic and the speeds of P and Q after impact are v_1 and v_2 respectively. Given that $\alpha = \frac{m_2}{m_1}$

(i) Determine the value of α if $u_1 = 20v_2$

(ii) Show that the fraction of energy lost by P is $\frac{4\alpha}{(1+\alpha)^2}$

Solution

(i) $m_1u_1 = m_1v_1 + m_2v_2$

$m_1(u_1 - v_1) = m_2v_2$

$(u_1 - v_1) = \alpha v_2$..(1)

${}^1\!/_2\, m_1u_1^2 = {}^1\!/_2\, m_1{v_1}^2 + {}^1\!/_2\, m_2{v_2}^2$

$m_1(u_1^2 - v_1^2) = m_2({v_2}^2\ )$

$(u_1^2 - v_1^2) = \alpha v_2^2$----------------------[2]

equating [2] ÷ [1] : $\frac{\alpha(u_1^2 - v_1^2)}{\alpha(u_1 - v_1)} = \frac{\alpha v_2^2}{\alpha v_2}$

$$\frac{(u_1 - v_1)\ (u_1 + v_1)}{(u_1 - v_1)} = \frac{v_2^2}{v_2}$$

$(u_1 + v_1) = v_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$

(1) + (3): $2u_1 = \alpha v_2 + v_2$

$u_1 = \frac{(1+\alpha)}{2}\ v_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots. (4)$

but $u_1 = 20v_2$

$$20v_2 = \frac{(1+\alpha)}{2}\ v_2$$

$$\alpha = 39$$

(iii) k.e of p before collision= $^1/_2\ m_1u_1^2$

k.e of p after collision= $^1/_2\ m_1v_1^2$

energy lost = $^1/_2 m_1u_1^2 - {^1/_2}\ m_1v_1^2$

fraction of energy lost = $\frac{^1/_2 m_1u_1^2 - {^1/_2} m_1v_1^2}{^1/_2 m_1u_1^2}$

fraction of energy lost = $\frac{(u_1^2 - v_1^2)}{u_1^2} = \frac{(u_1 - v_1)(u_1 + v_1)}{u_1^2}$

from (i) above $(u_1 + v_1) = v_2$, $(u_1 - v_1) = \alpha v_2$

$$u_1 = \frac{(1+\alpha)}{2}\ v_2$$

fraction of energy lost = $\frac{(\alpha v_2)(v_2)}{\left[\frac{(1+\alpha)}{2}v_2\right]^2} = \frac{4\alpha}{(1+\alpha)^2}$

3. A body explodes and produces two fragments of masses m and M. If the velocities of the fragments are u and v respectively, show that the ratio of kinetic energies of the fragments is

$$\frac{E_1}{E_2} = \frac{M}{m}$$

Where E_1 is the kinectic energy of m and E_2 is the kinectic energy of M

Solution

$$E_1 = \frac{1}{2}mu^2 \qquad and \qquad E_2 = \frac{1}{2}Mv^2$$

By law of conservation of linear momentum :

$$mu = -Mv$$

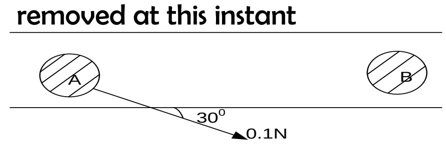
$$\therefore v = \frac{-mu}{M}$$

$$E_2 = \frac{1}{2}M\left(\frac{-mu}{M}\right)^2 = \frac{1}{2}\frac{m^2u^2}{M}$$

$$\frac{E_1}{E_2} = \frac{\left(\frac{1}{2}mu^2\right)}{\left(\frac{1}{2}\frac{m^2u^2}{M}\right)} = \frac{M}{m}$$

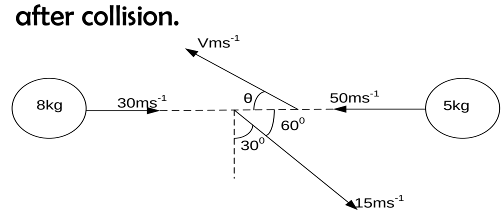
Exercise:13

1. A 4kg ball moving at 8m/s collides with a stationery ball of mass 12kg, and they stick together. Calculate the final velocity and the kinetic energy lost in impact **An [2m/s, 96J]**
2. A body of mass **m** makes a head on , perfectly elastic collision with a body of mass **M** initially at rest. Show that $\frac{\Delta E}{E_0} = \frac{4\left(\frac{M}{m}\right)}{\left(1+\frac{M}{m}\right)^2}$ where $E_0$is original kinetic energy of the mass **m** and ΔE the energy it loses in the collision
3. A metal sphere of mass m_1, moving at velocity u_1 collides with another sphere of mass $m_2$moving at velocity $u_2$in the same direction. After collision the spheres stick together and move off as one body. Show that the loss in kinetic energy E during collision is given by
$$\boldsymbol{E = \frac{\beta(u_1 - u_2)^2}{2(m_1 + m_2)}\ \ where\ \beta = m_1m_2}$$
4. A stationary radioactive nucleus disintegrates into an α −particle of relative atomic mass 4, and a residual nucleus of relative atomic mass 144. If the kinetic energy of the α −particle is 3.24x10^{-13}J, what is the kinetic energy of the residual nucleus **An(9x10^{-15}J)**
5. The diagram below shows a body A of mass 2kg resting in a frictionless horizontal gully in which it is constrained to move. It is acted upon by a force shown below for 5s after which time it strikes and sticks to the body B of mass 3kg, the force being removed at this instant



what will the speed of the combined masses be. **An(0.087m/s)**

6. Two balls collide and bounce off each other as shown below. Determine the final velocity v of 5kg mas if 8kg mass has a speed of $15ms^{-1}$ after collision.



7. An alpha particle of mass 4 units is incident with a velocity u on a stationary helium nucleus of equal mass. After collision, an alpha particle moves with a velocity $\frac{u}{2}$ at an angle of 60° to its initial direction while the helium nucleus moves at angle θ to the initial direction of the alpha particle. Calculate the velocity of the helium nucleus after collision and the value of θ. An($\frac{u\sqrt{3}}{2}\boldsymbol{ms^{-1}},\ \boldsymbol{\theta = 30°}$)

Application of law of conservation of momentum

(a) Consider a horse pipe of cross-sectional area A giving a water jet of velocity v, if the water hits the wall and comes to rest then;

Force due to water $= mass\ per\ second\ x\Delta velocity$

But $mass\ per\ second = densityxvolume\ per\ second$

$mass\ per\ second = \rho xAreaxheight\ per\ second$

$mass\ per\ second = \rho Axvelocity$

$mass\ per\ second = \rho vA$

Force $= \rho vAx\Delta velocity$

$\boxed{\text{Force } = \rho Av^2}$

Examples

1. Water leaves horse pipe at a rate of 5.0kgs^{-1} with a speed of 20ms^{-1} and is directed horizontally on a wall which stops it. Calculate the force exerted by the water on the wall.

   Solution

   Force due to water= $mass\ per\ second\ xvelocity\ change = 5\ x(20-0) = 100N$

2. A helicopter of mass 1.0x10^3kg hovers by imparting a downward velocity v to the air displaced by its rotating blades. The area swept pout by the blades is 80m^2. Calculate the value of v. (density of air=1.3kgm^{-3})

   Solution

   $$F = \rho Av^2$$
   $$mg = \rho Av^2$$
   $$1.0x10^3x9.81 = 80xvx1.3x(v-0)$$
   $$1.0x10^3x9.81 = 104v^2$$
   $$v = 9.8m/s$$

3. Sand falls onto a conveyor belt at a constant rate of 2kgs^{-1}. The belt is moving horizontally at 3ms^{-1}. Calculate
   (a) The extra force required to maintain the speed of the belt
   (b) Rate at which this force is dong work
   (c) The rate at which the kinetic energy of the sand increases

   Solution

   Force =mass per second $x$velocity change
   $= 2x3\ = 6N$

   Rate of doing work =force x velocity change
   $= 6x3 = 18Js^{-1}$

   Rate of k.e $= \frac{1}{2}m\ x\ (velocity\ change)^2$
   $= \frac{1}{2}x2x3^2 = 9Js^{-1}$

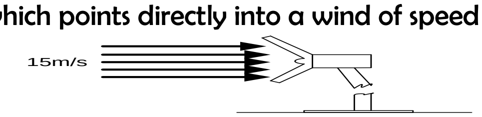
Exercise:14

1. A horizontal jet of water leaves the end of a hose pipe and strikes a wall horizontally with a velocity of 20m/s. If the end of the pipe has a diameter of 2cm, calculate the force that will be exerted on the wall. **An(125.7N)**

2. An astronaut is outside her space capsule in a region where the effect of gravity can be neglected. She uses a gas gun to move herself relative to the capsule. The gas gun fires gas from a muzzle of area 1.60mm^2 at a speed of 150ms^{-1}. The density of the gas is 0.800kgm^{-3} and the mass of the astronaut including her space suit is 130kg. calculate
   (a) The mass of gas leaving the gun per second
   (b) The acceleration of the astronaut due to gun, assuming that the change in mass is negligible

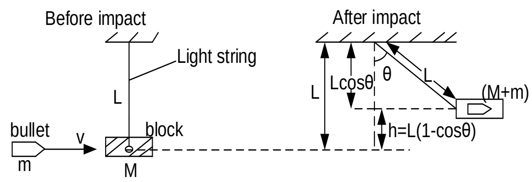
   An(1.92x$10^{-2}$$kgs^{-1}$, 2.22x$10^{-2}$$ms^{-2}$)

3. The blades of a large wind turbines, designed to generate electricity, sweeps pout an area of 1400m^2 and rotates about a horizontal axis which points directly into a wind of speed 15m/s



   (a) Calculate the mass of air passing per second through the area swept out by the blades (take the density of air to be 1.2 kg/m^3)
   (b) The mean speed of the on the far side of the blades is reduced to 13m/s. how much kinetic energy is lost by the air per second **An(2.5x10^4kg/s, 7.1x10^5J/s))**

4.1.9: BALLISTIC PENDULUM



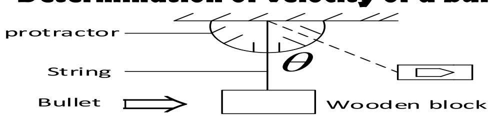
Resolving along the vertical gives $Lcos\theta$

But $L = Lcos\theta + h$

$h = L - Lcos\theta = L(1 - cos\theta)$

The device illustrates the laws of conservation of momentum and mechanical energy

Determination of velocity of a bullet using a ballistic pendulum

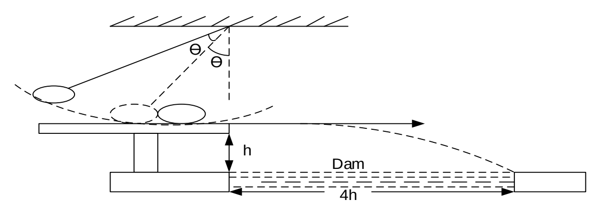


- ❖ A massive wooden block of known mass M is suspended from a fixed point by a string freshly blackened with charcoal.
  - ❖ A protractor is fixed at the point of the suspension of the string as shown above
- ❖ A bullet of mass m is fired from close range so that it gets embedded in the block and the first angle of swing θ from the vertical is read and noted.
- ❖ The length l of the string is measured and recorded.
- ❖ The velocity of the bullet u is obtained from

$$u = \left(\frac{m + M}{m}\right)\sqrt{2gl(1 - cos\theta)}$$

Exercise 15

1. A block of wood of mass 1.00kg is suspended freely by a thread. A bullet of mass 10g is fired horizontally at the block and becomes embedded in it. The block swings to one suede rising a vertical distance of 50cm. with what speed did the bullet hit the block **An[319.4m/s]**
2. A circular ring is tied to a roof using a string of length, l and displaced such that it makes an angle of 2θ with the vertical, where $\theta = 30°$. It is then released to throw a spherical ball horizontally across the dam at a height, h. It collides in elastically with the ball when at angle θ and move together until the ball leaves the bench horizontally to cross the dam of width 4h.



if the bench is frictionless and the masses are equal, show that $h = \frac{l(\sqrt{3}-1)}{32}$. Hence if $l = 128cm$,find the velociy with whoich the ball hits the ground

UNEB 2019 NO.1

(c) State Newtons laws of motion and use them to derive the laws of conservation of momentum. (06 marks)

(d) A body of mass 800kg moving at 30ms^{-1} collides with another body of mass 1400kg moving in the same direction at 25ms^{-1}. The two bodies stick together after collision. Calculate the;.

(i) common velocity just after collision **An(**$26.82ms^{-1}$**)** (02 marks)

(ii) kinetic energy lost during collision **An(**$6,256.36J$**)** (03 marks)

UNEB 2018 NO.1c

(i) Explain why a passenger in a car jerks forwards when the brakes are suddenly applied. (03 marks)

(ii) Use Newton's second law to define the Newton. (04 marks)

UNEB 2017 NO.1

(a) (i) State Newton's laws of motion (03marks)

(ii) A molecule of gas contained in a cube of side l strikes the wall of the cube repeatedly with a velocity u. Show that the average force F on the wall is given by $F = \frac{mu^2}{l}$ where m is the mass of the molecule (04marks)

(b) (i) Define the **linear momentum** and state the **law of conservation of linear momentum**. (02marks)

(ii) A body of mass m_1 moving with a velocity u, collides with another body of mass m_2 at rest. If they stick together after collision, find the common velocity with which they will move (04marks)

UNEB 2013 No 3(a)

(I) State the law of conservation of linear momentum (01mark)

(II) A body explodes and produces two fragments of masses m and M. If the velocities of the fragments are u and v respectively, show that the ratio of kinetic energies of the fragments is

$$\frac{E_1}{E_2} = \frac{M}{m}$$

Where E_1 is the kinectic energy of m and E_2 is the kinectic energy of M (04marks)

UNEB 2011 NO.2

(a) State Newton's laws of motion (03marks)

(b) Use Newton's laws of motion to show that when two bodies collide their momentum is conserved (04marks)

(c) Two balls P and Q travelling in the same line in opposite directions with speeds of 6ms^{-1} and 15ms^{-1} respectively make a perfect inelastic collision. If the masses of P and Q are 8kg and 5kg respectively, find the

(i) The velocity of P (04marks)

(ii) Change in kinetic energy **An[v=2.08ms^{-1},278.38J]** (04marks)

(d) (i) what is an impulse of a force (01marks)

(ii)Explain why a long jumper should normally land on sand. (04marks)

UNEB 2008 NO 4

a) State

(i) Newton's laws of motion (03 marks)

(ii) The principle of conservation of momentum (01 mark)

b) A body A of mass M_1 moves with velocity U_1 and collides head on elasticity with another body B of mass M_2 which is at rest. If the velocities of A and B are V_1 and V_2 respectively and given that x= $\frac{m_1}{m_2}$ Show that;

i) $\frac{u_1}{v_1} = \frac{x+1}{x-1}$ (04 marks)

ii) $\frac{v_2}{v_1} = \frac{2x}{x-1}$ (03 marks)

UNEB 1997 No 2

a) Define the terms momentum [01marks]

b) A bullet of mass 300g travelling at a speed of 8ms^{-1} hits a body of mass 450g moving in the same direction as the bullet at 15ms^{-1}. The bullet and body move together after collision. Find the loss in kinetic energy [06marks]

c) i) State the work energy theorem [01mark]

ii) A ball of mass 500g travelling at a speed of 10ms^{-1} at 60^0 to the horizontal strikes a vertical wall and rebounds with the same speed at 120^0 from the original direction. If the ball is in contact with the wall for 8x10^{-3}s, calculate the average force exerted by the ball.

An [625N] [06marks]

4.1.10: FORCE

Force is anything which changes a body's state of rest or uniform motion in a straight line
The unit of force is **a newton**

4.1.11: CONSERVATIVE AND NON CONSERVATIVE FORCES

1. **A conservative force** is a force for which the work done in moving a body round a closed path is zero.

4.2.0: SOLID FRICTION

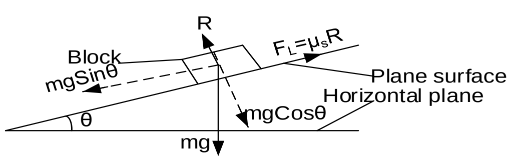
Friction is the force that opposes relative motion of two surfaces in contact.

4.2.3: Molecular explanation for occurrence of friction

- Surfaces have very small projections and when placed together the actual area of contact of two surfaces is very small, hence the pressure at points of contact is very high. Projections merge to produce welding and the welding's have to be broken for relative motion to occur. This explains the fact that friction opposes relative motion between surfaces in contact
- When the area between the surfaces is changed, the actual area of contact remains constant. Therefore no change in friction. This explains the fact that friction is independent of the area of contact provided normal reaction is constant
- Increasing normal reaction, increases the pressure at the welds. This increases the actual area of contact to support the bigger load, and hence a greater limiting frictional force . Therefore friction is proportional to normal reaction.

4.2.4: Measurement of coefficient of static friction

Method I



Method 2

- The mass m of the wooden block is determined and placed on a horizontal plane surface.
- A string is attached to the block and passed over a smooth pulley carrying a scale pan at the other end.
- Small masses are added to the scale pan one at a time, till the block just slides
- The total mass M of the scale pan and the masses added is obtained.
- Coefficient of static friction $\mu = \frac{M}{m}$

Measurement of Limiting friction

Alternatively

- The wooden block is placed on a horizontal plane surface.
- A string is attached to the block and passed over a smooth pulley carrying a scale pan at the other end.
- Small masses are added to the scale pan one at a time, till the block just slides
- The total mass M of the scale pan and the masses added is obtained.
- limiting friction $f = Mg$

CHAPTER 5: WORK, ENERGY AND POWER

5.1.0: Work

Definition

Work is defined as the product of force and distance moved in the direction of the force

or

Work done is also defined as the product of the component of the force in the direction of motion and displacement in that direction

Explain why it is easier to walk on a straight road than an inclined road up hill.

When walking on a level ground, work is done only against the frictional force. While when walking up hill, work is done against both frictional force and the component of the weight of the person along the plane of the hill.

Explain whether a person carrying a bucket of water does any work on the bucket while walking on a level road

bucket

5.2.0 : ENERGY

This is the ability to do work.

THE PRINCIPLE OF CONSERVATION OF ENERGY

It states that energy is neither created nor destroyed but changes from one form to another

5.2.2: WORK-ENERGY THEOREM

It states that the work done by the net force acting on a body is equal to the change in its kinetic energy.

Examples

1. A car mass 1000kg moving at 50ms^{-1} skid to rest in 4s under a constant retardation. Calculate the magnitude of the work done by the force of friction

Solution

a) Using $v = u + at$

$0 = 50 + 4a$

$a = -12.5m/s^2$

Frictional force $= ma$

$= 1000x - 12. = 12500N$

$S = ut + ½\, at^2$

$S = 50x4 + ½\, x - 12.5x4^2$

$S = 100m$

W= $FxS = 12500x100$

Work done $= 1.25x10^6J$

Alternatively

$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

W= $\frac{1}{2}x1000x50^2 - \frac{1}{2}\,x1000x0^2$

Work done $= 1.25x10^6J$

2. A bullet travelling at 150ms^{-1} will penetrate 8cm into a fixed block of wood before coming to rest. Find the velocity of the bullet when it has penetrated 4cm of the block.

Solution

$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = FxS$

$\frac{1}{2}mx0^2 - \frac{1}{2}mx150^2 = max0.08$

$a = -140625$ms^{-2}

Using $v^2 = u^2 + 2as$

$v^2 = 150^2 + 2x(-140625)x\frac{4}{100}$

$v = 106.06ms^{-1}$

THE PRINCIPLE OF CONSERVATION OF MECHANICAL ENERGY

States that in a mechanical system the total mechanical energy is a constant provided that no dissipative forces act on the system.

Examples of principle of conservation of M.E

i) A body thrown vertically upwards

Consider a body of mass m projected vertically upwards with speed u from a point on the ground.

At point A

At point B

K.E = ½ mv^2 and $P.E = mgx$

But $v^2 = u^2 - 2gx$

Since the total mechanical energy at all points is constant then the mechanical energy of a an object projected vertically upwards is conserved provided there is no dissipative force.

ii) A body falling freely from a height above the ground

Consider a body of mass 'm' at a height 'h' from the ground surface and at rest

At point A

$K.E = 0$ (at rest) and P.E $= mgh$

Total energy $= K.E + P.E = mgh$

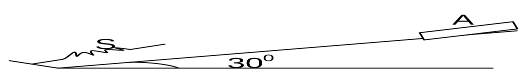
At point B

Total energy $= mgh$

Since the total mechanical energy at all points is constant then the mechanical energy of a freely falling object is conserved provided there is no dissipative force.

Examples

1. A block of mass 1kg is released from rest and travels down a rough incline of 30° to the horizontal a distance of 2m before striking a spring of force constant 100Nm^{-1}. The coefficient of friction between the block and the plane is 0.1

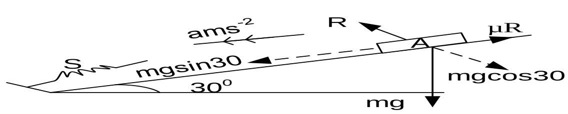


Calculate the:

(i) velocity of B just before it strikes the spring

(ii) maximum compression of the spring

solution



$$\boldsymbol{F = ma}$$

$$ma = mgsin30 - \mu R \text{ but } R = mgcos30$$

$$ma = mgsin30 - 0.1mgcos30$$

$$a = 4.055ms^{-2}$$

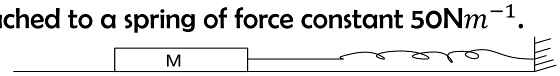
$$v^2 = u^2 + 2as$$

$$v = \sqrt{0^2 + 2x4.055x2} = 4.027ms^{-1}$$

$$(ii) \quad \frac{1}{2}ke^2 = \frac{1}{2}mv^2$$

$$e = \sqrt{\frac{1x(4.027)^2}{100}} = 0.4027m$$

1. The figure below shows a wooden block M of mass 990g resting on a rough horizontal surface and attached to a spring of force constant 50Nm^{-1}.



When a sharp nail of mass 10g shot at close range to the block, the spring is compressed by a distance of 20cm. If the work done against friction is $9x10^{-2}J$, Find the initial speed of the nail just before collision with the block.

Solution

After collision By conservation of energy: K.e of the nail and block=incresae in P.E+Work again friction

$$\frac{1}{2}(m + M)\, v^2 = \frac{1}{2}kx^2 + 9x10^{-2}J$$

$$\frac{1}{2}(0.01 + 0.99)\, v^2 = \left(\frac{1}{2}x50x0.02^2 + 9x10^{-2}J\right)$$

$$v = 0.0141m/s$$

Before collision: $m_1\,u_1 + m_2\,u_2 = (m_1 + m_2\,)v$

$$(0.01u) + 0.99x0 = (0.01 + 0.99)x0.0141$$

$$u = 1.41m/s$$

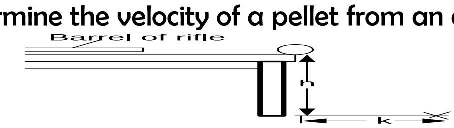
$$P = 2x57.7x0.5 = 57.7N$$

Exercise:17

2. A particle A of mass 2kg and a particle B of mass 1kg are connected by a light elastic string C and initially held at rest 0.9m apart on a smooth horizontal table with the string in tension. They are simultaneously released. The string releases 12J of energy as it contracts to its natural length.

Calculate the velocity acquired by each of the particles and find where the particles collide
An(2m/s, 4m/s, 0.3m from A)

3. A student devises the following experiment to determine the velocity of a pellet from an air rifle



A piece of plasticine of mass **M** is balanced on the edge of a table such that it just fails to fall off. A pellet of mass, **m** is fired horizontally into the plasticine and remains embedded in it. As a result the plasticine reaches the floor a horizontal distance k away. The height of the table is **h**

(i) show that the horizontal velocity of the plasticine with pellet embedded is $k\left(\frac{g}{2h}\right)^{1/2}$

(ii) obtain an expression for the velocity of the pellet before impact with the plasticine

4.

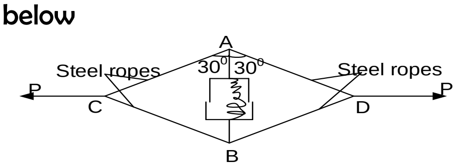
As shown in the diagram, two trolleys P and Q of mass 0.50kg and 0.30kg respectively are held together on a horizontal track against a spring which is in a state of compression.

(a) When the spring is released the trolley separate freely and P moves to the left with an initial velocity of 6m/s. calculate
   (i) Initial velocity of Q
   (ii) The initial total kinetic energy of the system

(b) Calculate the initial velocity of Q if trolley P is held still when the spring under the same compression as before us released

An(10m/s, 24J, 12.5m/s)

5. A muscle exerciser consists of two steel ropes attached to the ends of a strong spring contained in a telescopic tube. When the ropes are pulled sideways in opposite directions in the diagram below



The spring has an uncompressed length of 0.8m. the force F in newton required to compress the spring to a length x in meters is given by $F = 500(0.80 - x)$

The ropes are pulled with equal and opposite forces, P so that the string is compressed to a length of 0.60m and the ropes make an angle of 30^0 with the length of the springs

(a) Calculate the force F

(b) the work done in compressing the spring
   (i) by considering forces at A or B, calculate the tension in each rope
   (ii) by considering forces at C or D, calculate the force P **An(100N, 10J, 57.7N, 57.7N)**

5.3.0: POWER

It's the rate of doing work.

Its units are watts(W) or joule per second [Js^{-1}]

Power $= \frac{\text{Work done}}{\text{time}}$

P$= \frac{\text{F x d}}{\text{t}}$

P$= Fx\ \frac{d}{t}$

$P = Fxv$

PUMP RAISING AND EJECTING WATER.

Consider a pump which is used to raise water from a source and then eject it at a given speed. The total work done is sum of potential energy in raising the water and kinetic energy given to the water. The work done per second gives the rate (power) at which the pump is working.

$$\boxed{work\ done\ per\ second = P.E\ given\ to\ water\ per\ second + K.E\ given\ to\ water\ per\ second}$$

UNEB 2020 No.1

(a) (i) State the laws of friction. (03marks)

(ii) Use molecular theory to explain the laws stated in (a)(i) (06marks)

(b) Describe briefly how to measure limiting friction between a wooden block and a plane surface (04marks)

(c) A block of wood of mass 3.95kg rests on a horizontal table of height 5.0m at a distance of 6.0m from the edge of the table. A bullet of 50.0g moving with a horizontal velocity of 500ms^{-1} hits and gets embedded in the block. If the coefficient of dynamic friction between the block and the table is 0.3

(i) Find the initial velocity of the block after the collision with the bullet (02marks)

(ii) Calculate the horizontal distance from the table to the point where the block hits the ground

An $(i) = 6.25ms^{-1}, (ii) = 1.96m$ (05marks)

UNEB 2017 No.1c

A bullet of mass 10g moving horizontally with a velocity of 300m/s embedds into a block of wood of mass 290g which rests on a rough horizontal floor. After impact, the block and bullet move together and come to rest when the block has travelled a distance of 15m. Calculate the coefficient of slidding friction between the block and the floor. **An**(0.34) (07marks)

UNEB 2015 No.1

(a) (i) What is meant by **a conservative force** (01mark)

(ii) Give **two** examples of a conservative force (01mark)

(b) (i) State the law of conservation of **mechanical energy** (01mark)

(ii) A body of mass m, is projected vertically upwards with speed, u. Show that the law of conservation of mechanical energy is obeyed through its motion (05marks)

(i) Sketch a graph showing variation of kinetic energy of the body with time (01mark)

(c) (i) Describe an experiment to measure the coefficient of static friction (04marks)

(ii) State two disadvantages of friction (01marks)

(d) A bullet of mass 20g moving horizontally strikes and gets embedded in a wooden block of mass 500g resting a horizontal table. The block slides through a distance of 2.3m before coming to rest. If the coefficient of kinetic friction between the block and the table is 0.3, calculate the

(i) Friction force between the block and the table (02marks)

(ii) Velocity of the bullet just before it strikes the block (04marks)

An($1.53N$, $95.68m/s$)

UNEB 2010 No3

(c) i) State the laws of solid friction [03marks]

ii) With the aid of a well labeled diagram describe an experiment to determine the co-efficient of kinetic friction between the two surfaces. [05marks]

d) A body slides down a rough plane inclined at 30^{0} to the horizontal. If the co-efficient of kinetic friction between the body and the plane is 0.4. Find the velocity after it has travelled 6m along the plane.

An[4.25m/s] [05marks]

UNEB 2008 No2

a) i) state the laws of friction between solid surfaces [03marks]

ii) Explain the origin of friction force between two solid surfaces it contact. [03marks]

(i) Describe an experiment to measure the co-efficient of kinetic friction between two solid surfaces.

b) i) A car of mass 1000kg moves a long a straight surface with a speed of 20ms^{-1}. When brakes are applied steadily, the car comes to rest after travelling 50m. Calculate the co-efficient of friction between the surface and the types. **An**[$\mu = 0.408$] [04marks]

UNEB 2001 No1

a) i)State the principle of conservation of mechanical energy. [01mark]

ii)Show that a stone thrown vertically upwards obey, the principle in (c) throughout its upward motion. [04marks]

CHAPTER 6: STATICS

Is a subject which deals with equilibrium of forces $e.g$ the forces which act on a bridge.

6.1.0: Conditions necessary for mechanical equilibrium

When forces act on a body then it will be in equilibrium when;

1. The algebraic sum of all forces on a body in any direction is zero
2. The algebraic sum of moments of all forces about any point is zero

6.3.4: CENTRE OF MASS AND CENTER OF GRAVITY

Centre of mass: This is a point at which the whole mass of a body is considered to be concentrated.

Centre of gravity: This point where the resultant force on the body due to gravity acts.

DETERMINATION OF CENTRE OF GRAVITY OF AN IRREGULAR LAMINA

- Make three holes near the edge of the card board
- Suspend the cardboard from one hole and allow it to swing freely
- Hung a pendulum bob from the same point of suspension
- Trace the outline of the pendulum on the sheet
- Repeat the procedure above using the other holes.
- The point of intersection of the three outlines is the centre of gravity of the board

6.2.1: Moment of a force

This is the product of a force and the perpendicular distance of its line of action from the pivot.

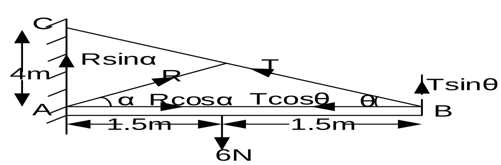
6.2.2: Principle of moments

It states that when a body is in mechanical equilibrium, the sum of clockwise moments about a point is equal to the sum of anticlockwise moments about the same point.

6.2.3: Beams hinged against the wall

1. A Uniform beam AB, 3.0m long and of weight 6N is hinged at a wall at A and is held stationary in a horizontal position by a rope attached to B and joined to a point C on the wall, 4.0m vertically above A. Find
   (i) the tension T in the rope
   (ii) the magnitude and direction of the Reaction R at the hinge.

   Solution



$R = 3.74N$

The reaction at A is 3.74 at 53.28° to the beam

2. A uniform beam AB of mass 20kg and length 2.4m is hinged at a point A in a vertical wall and is maintained in a horizontal position by means of a chain attached to B and to point C in a wall 1.5m above. If the bar carries a load of 10kg at a point 1.8m from A. calculate.
   i) The tension in the chain
   ii) The magnitude and direction of the reaction between the bar and the wall

   Solution

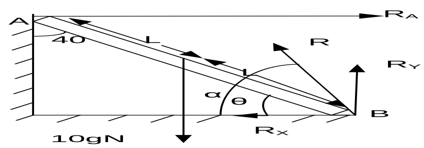
$R = 300.85N$

Reaction at A is 300.85 at 24.1° to the horizontal

6.2.4: Ladder problems

1. A uniform rod AB of mass 10kg is smoothly hinged at B and rests in a vertical plane with the end A against a smooth vertical wall. If the rod makes an angle of 40^o with the wall, find the reaction on the wall and the magnitude of the reaction at B

Solution

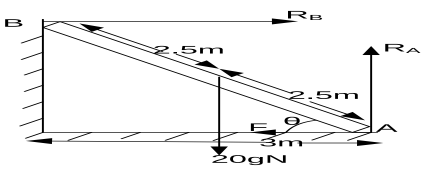


let length of the ladder be 2L

Reaction at B is $106.38N$ at $67.24°$ to the beam.

2. uniform ladder which is 5m long and has a mass of 20kg leans with its upper end against a smooth vertical wall and its lower end on a rough ground. The bottom of the ladder is 3m from the wall. Calculate the functional force between the ladder and the ground and the coefficient of friction

Solution



$Cos\theta = \frac{3}{5}$ $\quad\therefore\quad$ θ = 53.13^o

Resolving vertically: $R_A = 20gN$

$R_A = 20x9.81 = 196.2N$

$A^{\curvearrowright}$: $R_B\, x\, 5sin\theta = 20x9.81x2.5cos\theta$

R_B x $5sin53.13 = 20x9.81x2.5cos53.13$

$R_B = 73.56N$

Resolving horizontally: $R_B = F$

$F = 73.56N$

But $F = \mu x R_A$

$73.56 = \mu x196.2$

$\mu = 0.37$

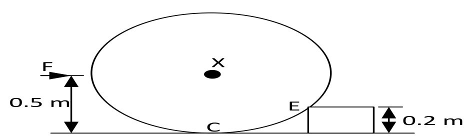
UNEB 2019 No 2

(i) Define moment of a force and give its S.I unit (2 marks)

(ii) Explain briefly how to locate the centre of gravity of an irregular sheet of cardboard. (4 marks)

(iii) State the conditions necessary for equilibrium of a rigid body under the action of a system of forces.(2 mks)

(iv) A wheel of radius 0.5 m rests on a level surface at point C and makes contact with edge E of a block of height 0.2 m as shown below



A force F is applied horiozotally through the axle of the wheel at X to just move the wheel over the block. If the weight of the wheel is 180N, find the;

(i) Force exerted at point E **An(300N)** (02marks)

(ii) Force F **An(239.9N)** (04marks)

(v) State the laws of solid friction and explain each of them (06marks)

UNEB 2018 No 2

(a) What is meant by **centre of mass?** (01mark)

(b) Explain why a long spanner is preferred to a shorter one in undoing a tight bolt. (03marks)

(c) A uniform ladder of length 10m and weight 400N, leans against a smooth wall and its foot rests on rough ground. The ladder makes an angle of $60°$ with the horizontal. If the ladder just slips when a person of wiehgt 800N climbs 6m up the ladder, calculate the;

(i) Reaction of the wall and the ground. (05marks)

(ii) Distance another person of weight 600N can climb so that the same reactions are exerted as in (c) (i) **An((i) 392.6N, 1262.6N at 71.9^o to the ground, (ii) 8.0m)** (02marks)

(d) (i) State the **principle of conservation of energy** (01mark)

(ii) How does the principle in (d) (i) apply to a child sliding down an incline? (02marks)

(e) A pump with power output of 147.1W can raise 2kg of water per second through a height of 5m and deliver it into a tank. Calculate the speed with which the water is delivered into the tank. (03marks)

Hint $\left(power\ x\ time = \frac{1}{2}mv^2 + mgh\right)$ **An(7.0 m/s)**

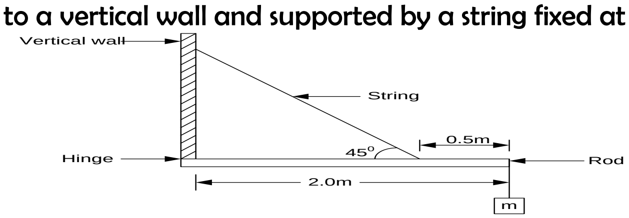
(f) Explain the effect of a couple on a rigid body. (03marks)

UNEB 2015 No 2

(a) **(i)** State the **principle of moments** (01mark)

(ii) Define the terms **center of gravity** and **uniform body** (2marks)

(b) The figure below shows a body, m of mass 20kg supported by a rod of negligible mas horizontally hinged to a vertical wall and supported by a string fixed at 0.5m from the other end of the rod



Calculate the

(i) Tension in the string (3marks)

(ii) Reaction at the hinge (3marks)

(iii) Maximum additional mass which can be added to the mas of 20 kg before the string can break given that the string cannot support a tension of more than 500N (2marks)

An(370N,270N,7.03kg)

UNEB 2006 No 2

(d) State the condition for equilibrium of a rigid body under the action of coplanar forces. (2mk)

(e) A 3m long ladder at an angle 60° to the horizontal against a smooth vertical wall on a rough ground. The ladder weighs 5kg and its centre of gravity is one third from the bottom of the ladder.

i) Draw a sketch diagram to show the forces acting on the ladder. (2mk)

ii) Find the reaction of the ground on the ladder. (4mk)

(Hint Reaction on the ladder =$\sqrt{R^2 + F^2}$) An(49.95N at 79.11° to the horizontal)

UNEB 2005 No2

(f) (i) Define centre of gravity (1 mark)

(ii) Describe an experiment to find the centre of gravity of a flat irregular card board. (3 marks)

UNEB 2000 No3

a) State the conditions for equilibrium of a rigid body under the action of coplanar forces.(2mk)

d) A mass of 5.0kg is suspended from the end A of a uniform beam of mass 1.0kg and length 1.0m. The end B of the beam is hinged in a wall. The beam is kept horizontal by a rope attached to A and to a point C in the wall at a height 0.75m above B

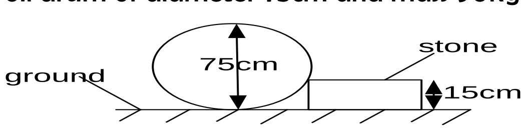
i. Draw a diagram to show the forces on the beam (2 marks)
ii. Calculate the tension in the rope (4 marks)
iii. What is the reaction exerted by the hinge on the beam (5 marks)

An (89.8N, 72.01N, at 3.95° to the beam)

UNEB 1998 No1

d) (i)Explain the term unstable equilibrium (3mk)

(ii) An oil drum of diameter 75cm and mass 90kg rests against a stone as shown



Find the least horizontal force applied through the centre of the drum, which will cause the drum to roll up the stone of height 15cm.

An(1177.2N) (5 marks)

CHAPTER 7: FLUID FLOW

A fluid element is a molecule of a fluid which follows the flow

A flowline is the path which an individual molecule in a fluid element describes

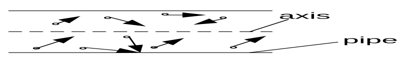
Why some fluids flow more easily than others

7.0 LAMINAR AND TURBULENT FLOW

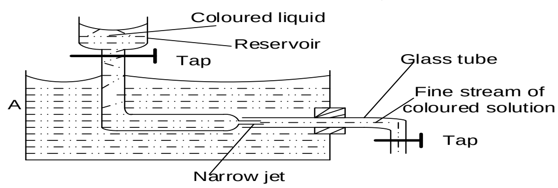
Laminar (steady/uniform) flow is the orderly flow of a liquid where flow lines are parallel to the axis of flow and equidistant layers from the axis of flow have the same velocity.

Laminar flow occurs at low velocities below the critical velocity.

Turbulent flow



7.1: EXPERIMENT TO DEMONSTRATE LAMINAR AND TURBULENT FLOW



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VISCOSITY

Viscosity is the frictional force between adjacent layers of a fluid.

Viscous drag is the frictional force experienced by a body moving in a fluid due to its viscosity.

7.2: Effects of temperature on viscosity

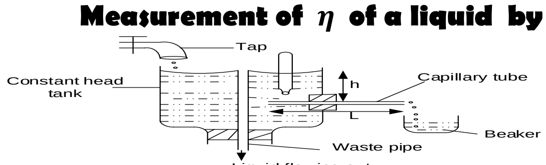
- In liquids, viscosity is due to intermolecular forces of attraction between layers moving at different speeds.
- In gases, viscosity is due to transfer of momentum .

7.3: COEFFICIENT OF VISCOSITY (η)

Coefficient of viscosity is the frictional force acting on a unit area of a fluid when in a region of unit velocity gradient **OR**

Coefficient of viscosity is the tangential stress which one layer of a fluid exerts on another layer in contact with it when the velocity gradient between the layers is $1s^{-1}$.

Measurement of η of a liquid by poiseuille's formula



- Measure and record the a constant head h .
- Measure and record volume V of liquid flowing through the capillary tube in time t
- Repeat several times by varying h to obtain a set of values for each volume v and calculate the volume per second$\left(\frac{V}{t}\right)$.
- Measure the length l of capillary tube, obtain the radius, r of capillary tube by measuring the mass of a known length of mercury column or by column travelling microscope method
- Plot a graph of $\left(\frac{V}{t}\right)$ against h and find the slope, s of the graph.
- Calculate the coefficient of viscosity η, from $S = \left(\frac{\pi r^4 \rho g}{8\eta l}\right)$

7.5: STOKE'S LAW AND TERMINAL VELOCITY

Stoke law states $F = 6\pi\eta\ rV$

F- viscous drag

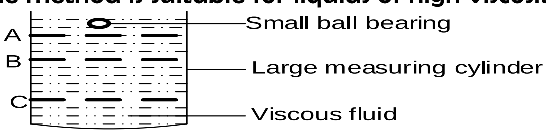
r-radius of the sphere

v- terminal Velocity of the sphere

η -Coefficient of viscosity of fluid

Measurement of η liquid by Stoke's law

The method is suitable for liquids of high viscosity such as glycerin and treacle



- Densities of the ball bearing and liquid ρ and σ respectively are obtained.
- Three reference marks A , B and C at equal distances are made on the sides of a tall transparent tube filled with the liquid.

Coefficient of viscosity is then calculated from Stoke's using

$\eta = \frac{2\,r^2\,g\,(\rho-\sigma)}{9\,V_0}$..................................[2]

7.6: TERMINAL VELOCITY

Terminal velocity is the maximum constant velocity attained by a body falling through a viscous fluid.

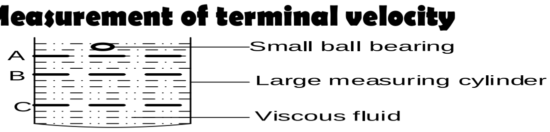
EXPLANATION OF TERMINAL VELOCITY

Consider a sphere of radius,r falling from rest through a viscous fluid.

- The forces acting on the sphere are its weight W downwards , up thrust upwards U due to the displaced fluid and the viscous drag, F upwards due to viscosity of the fluid.
- Initially $W > U + F$ and the sphere accelerates downwards. As its velocity increases, viscous drag increases and eventually $W = U + F$ and net force is zero and sphere moves with constant velocity.

A graph of velocity against time for an object falling in a fluid

Measurement of terminal velocity



- Densities of the ball bearing and liquid ρ and σ respectively are obtained.
- Three reference marks A , B and C at equal distances are made on the sides of a tall transparent tube filled with the liquid.

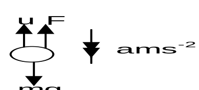
terminal velocity is obtained from

$V_o = \frac{AB}{t} = \frac{BC}{t} = \frac{AC}{2\,t}$......................[1]

Numerical examples

1. A spherical raindrop of radius 2.0x10^{-4}m, falls vertically in air at 20^{o}c, if the densities of air and water are 1.3kgm^{-3} and 1x10^{3}kgm^{-3} respectively and the viscosity of air at 20^{o}c is 1.8x10^{-5}pa. Find the terminal velocity of the drop

Solution



At terminal velocity : $Mg = U + F$

$\frac{4}{3}\pi r^3 \rho_s\, g = \frac{4}{3}\pi r^3 \rho_f\, g + 6\pi\,\eta\, rV_o$

$V_o = \frac{2\,r^2\,g\,(\rho_f-\rho_s)}{9\,\eta}$

$V_o = \frac{2x\,(2x10^{-4})^2 x9.81x\,(1x10^3-1.2)}{9x1.8x10^{-5}} = 4.84ms^{-1}$

2. Calculate the terminal velocity of a rain drop of radius 0.2cm. Density of water 1000kgm^{-3} and density of air 1kgm^{-3} and coefficient of viscosity of air is 10^{-3}Pa

Solution

$V_o = 8.7ms^{-1}$

Exercise:20

1. A small oil drop falls with terminal velocity of $4x10^{-4}ms^{-1}$ through air. Calculate the radius of the drop. What is the terminal velocity of oil drop if its radius is halved. (viscosity of air = $1.8x10^{-5}Nm^{-2}s$, density of oil = $900kgm^{-3}$, neglect density of air) **An** [$1.92x10^{-6}$m, $1.0x10^{-4}ms^{-1}$]
2. Calculate the terminal velocities of the following rain drops falling through air
   (a) One with a diameter of 0.3cm
   (b) One with a diameter of 0.01mm
   (density of water=$1000kgm^{-3}$, and viscosity of air=$1.0x10^{-3}$Pas. neglect air buoyancy)
   An [45 ms^{-1}, $5x\mathbf{10^{-4}}\ ms^{-1}$]
3. An explosion occurs at an altitude of 1000m where there is a constant horizontal wind speed of 10m/s. It is estimated that the smallest particles produced by the explosive have diameter of 0.01mm and density of $2000kgm^{-3}$. Calculate
   (a) The time taken for the smallest particles to fall to the ground
   (b) The horizontal distance travelled from the point of the explosion.
   (viscosity of air $1.8x10^{-5}$ Pas, density of air $1.2kgm^{-3}$)
   An [1.62$x10^5s$, 1620km]
4. Calculate the viscous drag on the drop of oil of radius 0.1mm falling through air at its terminal velocity. (viscosity of air $1.8x10^{-5}$Pas, density of air $850kgm^{-3}$) **An [$3.6x\mathbf{10^{-8}}\ N$]**

7.9: BERNOULLI'S PRINCIPLE

It states that for a non-viscous incompressible fluid flowing steadily, the sum of the pressure plus the potential energy per unit volume plus kinetic energy per unit volume is constant at all points on a stream line.

i.e. $\boxed{P + \tfrac{1}{2}\rho v^2 + \rho gh = a\ constant}$

P is the pressure with in the fluid
ρ is the density of the fluid
v is the velocity of the fluid
g is the acceleration due to gravity
h is height of the fluid (above reference line)

Derivation of Bernoulli's equation

Consider a tube of flow with in a non-viscous incompressible fluid of density ρ undergoing steady flow. If

P_1 and P_2 = pressure at X and Y respectively
V_1 and V_2 = velocities at X and Y espectively
h_1 and h_2 = Average heights at X and Y

work done per unit volume= $\frac{Forcex\ distance}{volume}$

$$\text{work done per unit volume} = \frac{PAxd}{Ad} = P$$

$K.E$ per unit volume= $\frac{1}{2}\frac{mv^2}{volume} = \frac{1}{2}\rho v^2$

$P.E$ per unit volume= $\frac{mgh}{volume} = \rho gh$

By conservation of energy

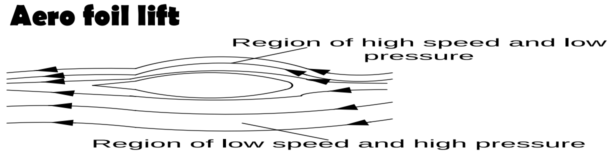
Work done by pressure difference = $\frac{Gain\ K.E}{volume} + \frac{Gain\ K.E}{volume}$

$$P_1 - P_2 = \left(\frac{1}{2}\rho\ {v_2}^2 - \frac{1}{2}\rho\ {v_1}^2\right) + (\rho gh_2 - \rho gh_1)$$

$$P_1 + \frac{1}{2}\rho\ {v_1}^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho\ {v_2}^2 + \rho gh_2$$

$P + \tfrac{1}{2}\rho v^2 + \rho gh$ =a constant

1. **Aero foil lift**



- An aero foil e.g. an air craft wing is shaped so that air flows <u>faster</u> along the top of the wings than below the wings.
- <u>By Bernoulli's principle</u> pressure below becomes <u>greater</u> than that above the wings.
- This <u>pressure difference</u> produces the <u>resultant force</u> called lift upwards force. It is this <u>force</u> which provides a force that lifts the plane off the ground at take off

Examples

1. A fluid of density 1000kgm^{-3} flows in a horizontal tube. If the pressure at the entry of the tube is 10^5Pa and at the exit is 10^3Pa, given that the velocity of the fluid at the entry is $8ms^{-1}$, calculate the velocity of the liquid at the exit.

Solution

$$P + ½\rho v^2 + \rho gh = a\ constant$$

$P_1 + ½\ \rho v_1^2 = P_2 + ½\ \rho v_2^2$ for horizontal flow

$$10^5 + ½\ x1000x8^2 = 10^3 + ½\ x1000xV_2^2$$

$$V_2 = 16.25ms^{-1}$$

2. Air flows over the upper surface of the wings of an aero plane at a speed of $81ms^{-1}$ and past the lower surfaces of the wings at $57ms^{-1}$. Calculate the lift force on the aero plane if it has a total wing area of $3.2m^2$. (density of air = $1.3kgm^{-3}$)

Solution

$P_1 + ½\ \rho v_1^2 = P_2 + ½\ \rho v_2^2$ for horizontal flow

$$P_2 - P_1 = \frac{1}{2}\rho(V_1^2 - V_2^2) = \frac{1}{2}x1.3x(81^2 - 57^2)$$

lift force, $F = (P_2 - P_1)A$

$$F = \left[\frac{1}{2}x1.3x(81^2 - 57^2)\right]x3.2 = 6.9x10^3N$$

7.12: FLUIDS AT REST

7.12.1: DENSITY AND RELATIVE DENSITY

Density of a substance is defined as the mass per unit volume of a substance.

$$\rho = \frac{m}{v}$$

S.I unit's kgm^{-3}

Relative density

Definition

It is the ratio of the density of a substance to density of an equal volume of water at 4°c

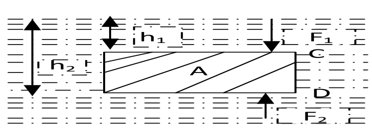
It is at 4℃ because water has maximum density of $1000kgm^{-3}$ at that temperature

7.12.2: ARCHIMEDE'S PRINCIPLE

It states that when a body is wholly or partially immersed in a fluid, it experiences an up thrust equals to the weight of the fluid displaced.

Verification of Archimedes' principle

Consider a rod of cross-sectional area A immersed in a large quantity of a fluid of density ρ such that its top level C, is h_1 meters below the surface of the fluid while its bottom level D, is h_2 meters as shown below



Vol of fluid displaced = vol of cylinder= $A(h_2 - h_1)$

Mass of fluid displaced = $A(h_2 - h_1)\rho$

Weight of fluid displaced = $A(h_2 - h_1)\rho g$..........(i)

Force at C: $F_1 = h_1\rho gA$

Force at D: $F_2 = h_2\rho gA$

Net Upward force ($Upthrust$) $= (h_2 - h_1)\rho gA$....(ii)

From equation (i) and equation (ii), therefore;

$Upthrust$ = weight of fluid displaced

Verification of Archimedes' principle using a spring balance.

- Fill the displacement can with water till water flows through the spout and wait until the water stops dripping.
- Weigh a solid object in air using a spring balance and record its weight W_a
- Place a beaker of known weight, W_b beneath the spout of the can.
- With the help of the spring balance, the solid object is carefully lowered into the water in the displacement can and wait until water stops dripping when it is completely immersed, its weight (apparent weight) is then read and recorded from the spring balance as W_w.
- Re weigh the beaker and the displaced water and record the weight as $W_{(b+w)}$
- If $(W_a - W_w) = (W_{(b+w)} - W_b)$, then Archimedes's principle is verified

Application of Archimedes' principle

It can be used to determine density and relative density of a solid and a liquid.

a) **Determination of density and relative density of a solid**

- Weigh a solid object in air using a spring balance and record its weight W_a.
- Immerse the solid object wholly in water and record the apparent weight W_w
- Weight of water displaced (up thrust in water) = W_a - W_w is calculated
- $R.D\ of\ the\ solid\ = \frac{Weight\ in\ air}{upthrus\ in\ water} = \frac{W_a}{W_a - W_w}$
- Density of solid = RD of solid x density of water

b) Determination of density and relative density of a liquid

- Weigh a solid object in air using a spring balance and record its weight W_a.
- Density of liquid = R.D of liquid x density of water

Law of floatation

It states that a floating body displaces its own weight in the fluid in which its floating.

Experiment to verify the law of floatation

- Pour water in a displacement can until it over flows through the spout and wait until the water stops dripping
- Place a beaker under the spout. Gently place an object which floats on water and wait until water stops dripping from the spout
- Displaced water is is collected in a beaker
- Determine the weight of the floating object and the weight of the displaced water
- The two weights are found to be the same, hence law of floatation

7.13: PRESSURE

Pressure is the force acting normally per 1m^2 area

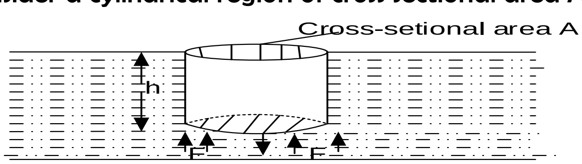
$$P = \frac{F}{A}$$

PRESSURE IN FLUIDS

The pressure in a fluid increase with depth, and all points at the same depth in the fluid are at the same pressure.

RELATION OF PRESSURE P WITH DEPTH h

Consider a cylindrical region of cross sectional area A and height h in a fluid of density ρ



The top of the cylinder is at the surface of the fluid and the vertical forces acting on it are its weight (mg) and an upward force F due to pressure p at the bottom of the cylinder.

$P = h\rho g$

UNEB 2020 Q 4

(a) Define the following
- (i) Pressure (01mark)
- (ii) Relative density (01mark)

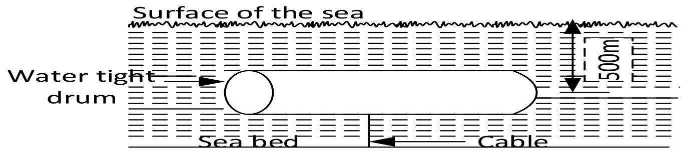
(b) (i) State Archimedes Principle. (01mark)

(ii) Describe an experiment to determine the relative density of a liquid. (04marks)

(c) **(i)** Derive the expression for **Bernoulli's equation** (05marks)

(ii) Explain why a person standing by the road side may be pulled towards the road when a fast moving bus passes by (03marks)

(d) A water tight drum tied to a cable anchored on the sea bed floats 500m beneath rhe sea surface



If the weight of the drum is 500N and its volume is $25m^3$, calculate the
- (i) Pressure on the drum due to the sea water **An[= $4.91x10^6Nm^{-2}$]** (02marks)
- (ii) Tension in the cable assuming it is vertical **an [= $2.25x10^6N$]** (03marks)

UNEB 2019 Q 4

(a) State and illustrate Archimedes principle (05marks)

(b) (i) State the law of floatation . (01mark)

(ii) Describe an experiment to verify the law in (b) (i) (05marks)

(c) **(i)** Write **Bernoulli's principle** and define each term in the equation (02marks)

(ii) Explain the origin of the lift force on the wing of a plane (03marks)

(iiii) Air flows over the upper surface of the wings of an aero plane at a speed of $120ms^{-1}$ and past the lower surfaces of the wings at $110ms^{-1}$. Calculate the lift force on the aero plane if it has a total wing area of $20m^2$. (density of air = $1.2kgm^{-3}$) **[an= $2.97x10^4N$]** (04marks)

UNEB 2016 Q.4

(a) **(i)** What is meant by **fluid element** and **flow line** as applied to fluid flow (02marks)

(ii) Explain why some fluids flow more easily than others. (03marks)

(b) **(i)** State **Bernoulli's principle** (01mark)

(ii) Explain how a pitot static tube works (04marks)

(c) Air flowing over the upper surface of an air craft's wings causes a lift force of 6400N. The air flows under the wings at a speed of 120m/s over an area of $28m^2$. Find the speed of air flow over an equal area of the upper surface of the air of the air craft's wings. (density of air= $1.2kgm^{-3}$) **An** $121.6ms^{-1}$ (4marks)

(d) **(i)** What is meant by **surface tension** and **angle of contact** of a liquid (02marks)

(ii) A water drop of radius 0.5cm is broken up into other drops of water of radius 1mm. Assuming isothermal conditions, find the total work done to break up the water drop. **An** $8.8x10^{-5}J$ (04marks)

UNEB 2014Q.4

(a) Define coefficient of viscosity and state its units (02marks)

(b) Explain the origin of viscosity in air and account for the effect of temperature on it (05marks)

(c) Describe, stating the necessary precautions an experiment to measure the coefficient of viscosity of a liquid using Stoke's law (07marks)

(d) A steel ball bearing of diameter 8.0mm falls steadily through oil and covers a vertical height of 20.0cm in 0.56 s. if the density of the steel is $7800kgm^{-3}$ and that of oil is 900 kgm^{-3}. Calculate:

- (i) Up thrust on the ball **An** $2.37x10^{-3}N$ (03 marks)
- (ii) Viscosity of oil **An** $0.674Nsm^{-2}$ (03 marks)

UNEB 2002 Q 3

a) i) Show that the weight of fluid displaced by an object is equal to the up thrust on the object. (5mks)

ii) A piece of metal of mass $2.60x10^{-3}kg$ and density $8.4x10^3kgm^{-3}$ is attached to a block of wax of mass $1.0x10^{-2}$ kg and density $9.2x10^2kgm^{-3}$. When the system is placed in a liquid, it floats with wax just submerged. Find the density of liquid. (04marks)

b) Explain the
- i) Terms laminar flow and turbulent flow (04marks)
- ii) Effects of temperature on the viscosity of liquids and gases (03marks)

c) i) Distinguish between static pressure and dynamic pressure (02marks)

CHAPTER 8: MECHANICAL PROPERTIES OF MATTER

Terms used

1. **Elasticity:** This is the ability of the material to regain its original shape and size when the deforming load has been removed.

2. **Elastic material:** This is a material which regains its original shape and size when the deforming load has been removed. E.g. Rubber band, spring.

3. **Elastic limit:** This is the **maximum load** which a material can experience and still regain its original size and shape once the load has been removed.
The elastic limit sometimes coincides with the limit of proportionality.

4. **Proportional limit:** This is the **maximum load** a material can experience for which the extension created on it is directly proportional to the load applied.

5. **Hooke's law:** it states that; the extension of a wire or spring is directly proportional to the applied load provided the proportional limit is not exceeded.

6. **Yield point:** This is a point at which there is a marked increase in extension when the stress or load is increased beyond the elastic limit.

7. **Plastic deformation:** This is when a material cannot recover its original shape and size when the deforming load has been removed.

8. **Ductility:** It is the ability of the material to be permanently stretched. or it is the ability of the material to be stretched appreciably beyond elastic limit. It can be drawn into different shapes without breaking.

Tensile stress and Tensile strain

Tensile stress: it is force acting per unit area of cross-section of a material.

Stress $= \frac{F}{A}$

Tensile strain: it is the extension per unit original length of the material.

Strain $= \frac{e}{L}$

Strain has no units because it is a ratio of two similar units

Examples

1. A metal bar has a circular cross section of diameter 20mm. If the maximum permissible tensile stress is $8x10^7$Nm^{-2}, calculate the maximum force which the bar can withstand.

Solution

Force$= stress\ xArea = 8x10^7 x\frac{\pi d^2}{4}$

$F = 8x10^7 x\frac{\left[\frac{22}{7}x(20x10^{-3})^2\right]}{4} = 2.513x10^4N$

2. A metal bar is of length 2.0m and has a square cross-section of side 40mm. When a tensile force of $80kN$ is applied, it is extends by 0.046mm, calculate
 (i) Stress (ii) Strain in specimen

Solution

stress$= \frac{Force}{Area} = \frac{80x1000}{(40x10^{-3})^2} = 5.0x10^7Nm^{-2}$

strain$= \frac{e}{l} = \frac{0.046x10^{-3}}{2} = 2.3x10^{-5}$

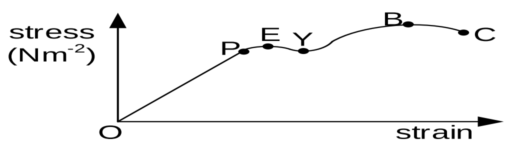
8.0: Experiment to study elastic properties of steel

- Two long, thin identical steel wires are suspended besides each other from the same rigid support B
- The wire P is kept taut and free of kinks by weight attached to its end
- The original length l of test wire Q is measured and recorded.
- The mean diameter d of test wire is determined and cross-sectional area $A = \frac{\pi\, d^2}{4}$ is found.
- Known weight, W is added to the free end of test wire and the corresponding extension e is read from the vernier scale.
- The procedure is repeated for different weights and for each extension, the load is removed to ensure that the wire goes back to the original length
- Results are tabulated including values of tensile stress $\left(\frac{W}{A}\right)$ and tensile strain $\left(\frac{e}{L}\right)$
- The graph of tensile stress versus tensile strain is plotted as below.

8.1: Stress-strain graphs

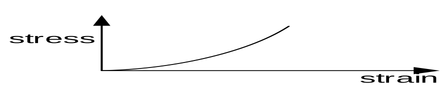
1. **Ductile material** e.g. copper, steel, iron

A ductile material is one which can be permanently stretched



P-Proportionality limit
E-Elastic limit
Y-Yield point
B-Breaking stress
C-Breaking point

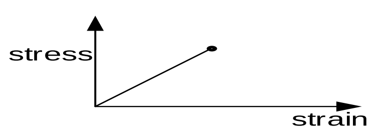
2. **Rubber**



Rubber does not obey Hooke's law except for a smaller load. This is because rubber has coiled molecules which uncoil when stretched

3. **Brittle materials** e.g. glass, chalk, rocks and cast iron

These are materials that can not be permanently stretched. It breaks as soon as the elastic limit has been reached



Brittle materials have only a small elastic region and do not under go plastic deformation. This behavior in glass is due to the existence of cracks in its surface. The high concentration of the stress at the crack makes the glass break.

8.3: Young's modulus (Modulus of elasticity)

Young's modulus is the ratio of tensile stress to tensile strain of a material

Young modulus $= \frac{stress}{strain}$ | $E = \frac{F}{A} \Big/ \frac{e}{L}$ | $E = \frac{F\,L}{A\,e}$

A is area, L is original length, e is extension

Examples

1. Find the maximum load which may be placed on steel of diameter 1mm if the permitted strain must not exceed 0.001 and young's modulus for steel is $2x10^{11}$Nm^{-2}

Solution

Stress$= Ex\ strain\ = 2x10^{11}x0.001 = 2x10^{8}\ Nm^{-2}$

Force$= stress\ xArea = 2x10^{8}x\frac{\pi d^2}{4}$

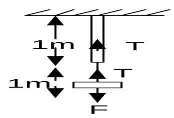
$F = 2x10^{8}x\frac{\left[\frac{22}{7}x(1x10^{-3})^2\right]}{4} = \ 1.571x10^{2}N$

2. A cylindrical copper wire and a cylindrical steel wire, each of length 1m and having equal diameter are joined at one end to form a composite wire 2m long. This composite wire is subjected to a tensile stress

until its length becomes 2.002m. Calculate the tensile stress applied to the wire (young modulus of copper $=1.2x10^{11}$Pa and Steel $=2x10^{11}$Pa)

Solution

[Recall from S.H.M wire in series experience the same tension and weight]



Total extension, $e = 2.002 - 2$

$e = 0.002m$

$e = e_1 + e_2$-------------[1]

Note: the two wires will experiences same stress

$$0.002 = e_1 + e_2$$

$$e = \frac{F\,L}{A\,E}$$

$$0.002 = \frac{F\,L_1}{A\,E_1} + \frac{F\,L_2}{A\,E_2}$$

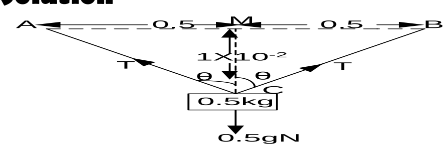
$$0.002 = \frac{F}{A}\left(\frac{1}{1.2x10^{11}} + \frac{1}{2x10^{11}}\right)$$

$$\frac{F}{A} = 1.5x10^8N$$

Stress $= 1.5x10^8N$

3. One The ends of a uniform wire of cross-sectional area 10^{-6}m^2 and negligible mass are attached to fixed points A and B which are 1m apart in the same horizontal plane. The wire is initially straight and outstretched. A mass of 0.5kg is attached to the mid point of the wire and hangs in equilibrium with the mid point at a distance 10mm below AB. Calculate the value of young's modulus for the wire

Solution



$CB^2 = 0.5^2 + (1x10^{-2})^2$

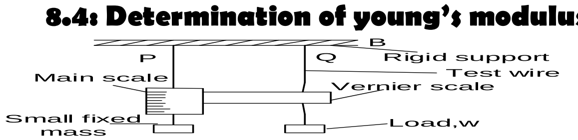
$CB = 0.5001m$

$AC = CB = 0.5001m$

Length ACB $= 0.5001\ x\ 2 = 1.0002m$

$$E = \frac{F\,L}{A\,e} = \frac{127.75\text{x}1}{10^{-6}\text{x}2\text{x}10^{-4}} = 6.39x10^{11}Nm^{-2}$$

8.4: Determination of young's modulus (Searle's apparatus)



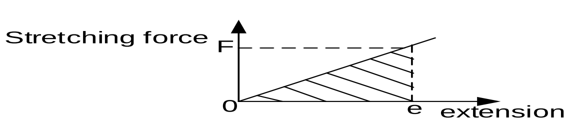
- Two long, thin identical steel wires are suspended besides each other from the same rigid support B
- The wire P is kept taut and free of kinks by weight A attached to its end
- A graph of weight W against extension e is plotted and its slope (s) obtained.
- Young's modulus is obtained from $E = \frac{S\,L}{A}$

Precautions

8.5: Energy stored in a stretched material [strain energy]

Consider a material of an elastic constant k, stretched by a force,F to extend by e.

By Hooke's law, the extension is directly proportional to the applied force provided the elastic limit is not exceeded.



Work done = area under the graph

Work done $= \frac{1}{2}F\,e$

But $F = ke$

Work done $= \frac{1}{2}k\,e^2$

The work done to stretch the material is stored as elastic potential in the material

Energy stored $= \frac{1}{2}k\,e^2$

Or Energy stored $= \frac{1}{2}F\,e$

By calculus [integration]

Examples

1. Calculate the energy stored in 2m long copper wire of cross-sectional area 0.55mm^2, if a force of 50N is applied to it

Solution

$$e = \frac{F\,L}{A\,E}$$

Energy stored = ½ Fe

$= \frac{1}{2} x 50 x \frac{2.8x0.1}{1.2x10^{11}x0.5x10^{-6}}$

$= 0.04J$

2. An elastic string of cross-sectional area 4mm^2 requires a force of 2.8N to increase its length by one tenth. Find young's modulus for the string if the original length of the string was 1m, find the energy stored in the string when it is extended.

Solution

Exercise: 24 [use g = 10ms^{-2}]

1. A metal column shortens by 0.25mm when a load of $120kN$ is placed upon it. Calculate;
   a) Energy stored in the column
   b) Loss of gravitational potential energy.
   An[15J, 30J]
2. A uniform steel wire of density 7800kgm^{-3} weighs 26g and 250cm long, it lengthens by 1.2mm, when stretched by a force of 80N, calculate;
   a) The value of young's modulus for steel
   b) The energy stored in the wire
   (**Hint volume** $=$ **Al** $= \frac{mass}{density}$) **An (2.03x10^{11}Nm^{-2}, 0.048J)**
3. A gymnast of mass 70kg hangs by one arm from high bar. If the gymnasts whole weight is assumed to be taken by the humerous bore (in the upper arm), calculate the stress in the humerous if it has a radius of 1.5cm **An (9.9x10^{5}Pa)**
4. Find the maximum load that can be support by a steel cable 1.5cm in diameter without its elastic limit being exceeded when the load is
   (a) In air (b) immerse in water
   An (1.41x10^{4}kg, (ii)depends on density of load)
5. A rubber cord a catapult has a cross-sectional area of 1.0mm^2 and un stretched length 10.0cm. It is stretched to 15cm and then released to project a missile of mass 5.0g. Calculate;
   (a) the energy stored in the rubber.
   (b) The velocity of projection
   (c) The maximum height that the missile could reach. (young's modulus for rubber=5.0x10^{8}Pa)
   An(6.25J, 50m/s, 125m)

8.7: FORCE ON A BAR DUE TO THERMAL EXPANSION OR CONTRACTION

When a bar is heated and then prevented from contracting as it cools, a force is exerted at the ends of a bar.

Consider a metal of young's modulus E, cross sectional Area A at a temperature θ_2°c fixed between two rigid supports.

When the bar is cooled to a temperature θ_1°C, the bar can not contract hence there will be forces on the rigid support.

If α is the mean co-efficient of linear expansion then $L_\theta = L_0(1 + \alpha\theta)$

L_θ is length of the bar at temperature θ°c

L_0 is length of the bar at temperature 0°c

$L_2 = L_0(1 + \alpha\theta_2)$ i

$L_1 = L_0(1 + \alpha\theta_1)$ ii

Subtracting: $L_2 - L_1 = L_0\, \alpha\Delta\theta$

$\alpha\,\Delta\theta = \frac{L_2 - L_1}{L_0}$

But strain $= \frac{L_2 - L_1}{L_0}$

$\boxed{Strain = \alpha\Delta\,\theta}$

Stress = E x strain

$\frac{F}{A} = Ex\alpha\Delta\,\theta$

$\boxed{F = EA\alpha\Delta\,\theta}$

Coefficient of linear expansion α is defined as the fractional increase in length at0°C for every degree rise in temperature.

UNEB 2020 No2

(a) Define the following terms as applied to materials
   (i) Stress (1 mark)
   (ii) Young's Modulus (1 mark)

(b) The velocity of compressional waves travelling along a rod omade of material of Young's Modulus, E and density, ρ, is given by $V = \left(\frac{E}{\rho}\right)^{1/2}$. Show that the formula is dimensionally consistent. (02 marks)

(c) Derive an expression for the energy stored in a stretched wire within the elastic limit (03 marks)

(d) A uniform wire of length 2.49m is attached to two fixed points A and B, a horizontal distance 2m apart. When a 5kg mass is attached to the midpoint C of the wire, the equilibrium position of C is 0.75m below the line AB. Neglecting the weight of the wire and taking Young's Modulus for the material to be $2.0x10^{11}Nm^{-2}$., find the;

(i) Strain in the wire **An(**0.004016**)** (04marks)

(ii) Stress in the wire **An** ($8.03x10^{8}Nm^{-2}$) (02marks)

(iii) Energy stored in the wire **An(0.204J)** (04marks)

(e) (i) Sketch the stress-strain curve for glass and explain its shape. (02marks)

(ii) Why does glass break easily. (01mark)

UNEB 2018 No3

(i) What is meant by a

(i) Brittle material? (1 mark)

(ii) Ductile material? (1 mark)

(ii) Give **one** example of each of the materials in (a) above (1 mark)

(iii) Explain why bicycle frames are hollow. (02 marks)

(iv) **(i)** Sketch a labeled graph of stress against strain for a ductile material (02marks)

(ii) Explain the main features of the graph in (d) (i) (04marks)

(v) Derive the expression for the energy stored per unit volume in a rod of length, L, Young's modulus, Y, when stretched through distance, e. (04marks)

(vi) A load of 5kg is placed on top of a vertical brass rod of radius 10 mm and length 50cm. If Young's modulus of brass is $3.5x10^{10}Nm^{-2}$. Calculate the;

(i) Decrease in length (03marks)

(ii) Energy stored in the rod (02marks)

An[$(i) = \mathbf{2.23x10^{-6}m}, (ii) = \mathbf{5.47x10^{-5}J}$ **]**

UNEB 2017 No4

(a) (i) Define **elastic deformation** and **plastic deformation** (02marks)

(ii) Explain what is meant by work hardening (02marks)

(b) (i) Sketch using the same axes, stress-strain curves for a ductile material and rubber. (03marks)

(ii) Explain the features of the curve for rubber (03marks)

UNEB 2015 No2

(c) (i) Define **young's modulus** (01mark)

(ii) Explain the precautions taken in the determination of Young's modulus of a wire (06marks)

(a) Explain why a piece of rubber stretches much more than a metal wire of the same length and cross-sectional area (02marks)

UNEB 2006 No 3

a) i) Define stress and strain (2 marks)

ii) Determine the dimensions of young's modulus (3 marks)

b) Sketch a graph of stress versus strain for a ductile material and explain its features (6 marks)

c) A steel wire of cross-section area $1mm^2$ is cooled from a temperature of 60^oc to 15^oc, find the;

i. Strain (2marks)

ii. Force needed to prevent it from contracting young's modulus = $2x10^{11}Pa$, coefficient of linear expansion for steel = $1.1x10^{-5}K^{-1}$ (3 marks)

d) Explain the energy changes which occur during plastic deformation (4 marks) **Ans ($4.95x10^{-4}$, 99N)**

CHAPTER 9: CIRCULAR MOTION

This is the motion of the body with a uniform speed around a circular path of fixed radius about a center.

Terms used in circular motion

1. **Angular velocity (ω)**

This is the is the rate of change of the angle for a body moving in a circular path.

Or rate of change of angular displacement i.e $\boldsymbol{\omega} = \frac{\Delta\theta}{\Delta t}$

For large angles and big time intervals. $\boldsymbol{\omega} = \frac{\theta}{t}$

Angular velocity is measured in radians per second ($rads^{-1}$)

2. **Period T**

This is the time taken for the body to describe one complete are revolution

$T = \frac{Circumference[distance\ around\ acircle]}{velocity}$

$T = \frac{2\pi r}{v}$

But $v = r\,\omega$

$T = \frac{2\pi r}{\omega r}$

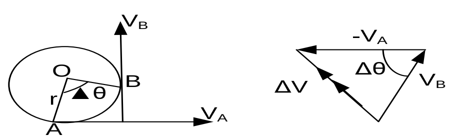
$\boxed{T = \frac{2\pi}{\omega}}$ units seconds.

3. **Acceleration**

Centripetal acceleration is defined as the rate of change of velocity of a body moving in a circular path and is always directed towards the centre.

9.1: Derivation of $a = \frac{v^2}{r}$

Consider a body of mass m moving around a circular path of radius r with uniform angular velocity ω and speed V. If initially the body is at point A moving with velocity V_A and after a small time interval Δt, the body is at point B where its velocity is V_B with the radius having moved an angle $\Delta\theta$



Vector form: $V_B - V_A = \Delta V$

Taking ΔV as arc of a circle:

$s = r\theta \Rightarrow \Delta V = V\Delta\theta$

$a = \frac{change\ in\ velocity}{time} = \frac{V\Delta\theta}{\Delta t}$

But $\frac{\Delta\theta}{\Delta t} = \omega = \frac{v}{r}$

$\boxed{a = \frac{v^2}{r}}$

EXERCISE:26

1. What force is required to cause a body of mass 3g to move in a circle of radius 2m at a constant rate of 4 revolutions per second. **An(3.8N)**
2. A helicopter 's rotor blades rotate such that the speed at the tip is $200ms^{-1}$. This is roughly the same for all helicopters regardless of the length of the blades. Calculate the frequency of rotation for the following;
   (i) Boeing Chinook-rotor balde length 9.14m,
   (ii) Siskorsky Black Hawk- rotor blade length 8.45m,
   (iii) Westland Lynx- rotor blade length 6.40m.

   Calculate also the maximum tension in (c) if the mass of the blades is 46kg. **An(3.5Hz, 3.8Hz, 5.0Hz,** $1.44x10^5$**N)**

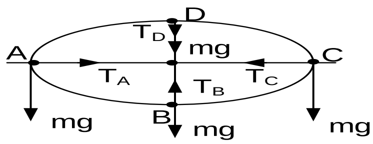
9.2: CENTRIPETAL FORCE

Centripetal force is an inward force towards the center of the circle required to keep a body moving in a circular path

$\boxed{F = \frac{m\,v^2}{r}}$ This is the expression for the centripetal force Or $\boxed{F = mr\,\omega^2}$

9.4: Motion in a vertical cycle

Consider a body of mass m attached to a string of length r and whirled in a vertical circle with a constant speed V. If there is no air resistance to the motion, then the net force towards the centre is the centripetal force.



At point A: $T_A = \frac{m\,v^2}{r}$ ------------------------(1)

The maximum tension in the vertical circle is experienced at B

T$_{max}$= $\frac{m\,v^2}{r}$ $+mg$- ----------------(2)

At point C: $T_C = \frac{m\,v^2}{r}$--------------------------(3)

The minimum tension is experienced on the top of the circle at point D

T$_{min}$= $\frac{m\,v^2}{r}$− mg------------------(4)

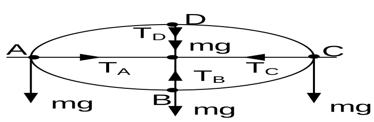
Note

If the speed of whirling is increased the string will most likely break at the bottom of the circle. Motion is tangential to the circle and when string breaks the mass will fly in a parabolic path.

Examples

1. An object of mass 3kg is whirled in a vertical circle of radius 2m with a constant sped of 12ms^{-1}, calculate the maximum and minimum tension in the string

Solution



Maximum tension is at B

$$T - mg = \frac{m\,v^2}{r}$$

T = $\frac{3x12^2}{2}$ $+3x9.81$ $= 245.43N$

Minimum tension is at D

T= $\frac{m\,v^2}{r} - mg$

$T = \frac{3x12^2}{2}$ $- 3x9.81$

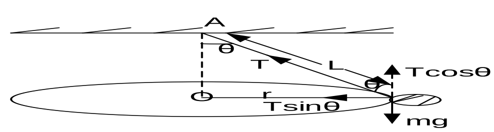
T $= 186.57N$

2. A stone of mass 800g is attached to string of length 60cm which has a breaking tension of 20N. The string is whirled in a vertical circle the axis of rotation at a height of 100cm from the ground.
   i) What is the angular velocity where the string is most likely to break?
   ii) How long will it take before the stone hits the ground?
   iii) Where the stone hit the ground

Solution

9.5: Motion in a horizontal circle [conical pendulum]

Consider a body of mass , m tied to a string of length L whirled in a horizontal circle of radius r at a constant speed V



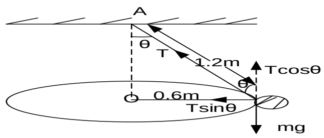
Explain why a mass attached to a string rotating at a constant speed in a horizontal circle will fly off at a tangent if the string breaks

- When a mass is whirled in a horizontal circle, the horizontal component of the tension ($Tsin\theta$) provides the necessary centripetal force which keeps the body moving in a circle without falling off.
- When the string breaks, the mass will not have any centripetal force and will continue in a straight line along the tangent.

Examples

1. A stone 0.5kg is tied to one end of a string 1.2m long and whirled in a horizontal circle of diameter 1.2m. Calculate;
   i) The length in the string ii) The angular velocity iii) The period of motion

Solution



i) $(\uparrow) T\cos\theta = 0.5gN$ -------(1)

But $\sin\theta = \frac{0.6}{1.2}$ ∴ θ= 30°

put into: (1) $T\cos 30 = 0.5x9.81$

$T = 5.60N$

ii) Angular velocity

$\omega = \sqrt{\frac{g}{L\cos\theta}} = \sqrt{\frac{9.81}{1.2\cos 30}}$

ω =3.07rads^{-1}

iii)Period , $T = \frac{2\pi}{\omega} = \frac{2x\frac{22}{7}}{3.07} = 2.05s$

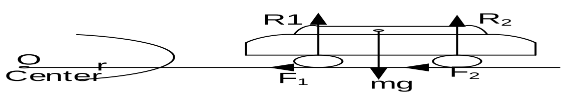
2. A body of mass 4kg is moving with a uniform speed 5ms^{-1} in a horizontal circle of radius 0.3m, find:
   i) The angle the string makes with the vertical
   ii) The tension on the string

Solution

9.6: Motion of a car round a flat track [negotiating a bend]

Consider a car of mass m moving round a circular horizontal arc of radius r with a speed v

A) Skidding of the car



$(\uparrow)$: $R_1 + R_2$ =mg----------------- (1)

$(\rightarrow)$: $F_1 + F_2 = \frac{m\,v^2}{r}$ -----------------(2)

The frictional forces F_1 and F_2 provide the necessary centripetal force

But $F_1 = \mu R_1$, $F_2 = \mu R_2$

$\mu\,(R_1 + R_2) = \frac{m\,v^2}{r}$ --------------------- (3)

(1) into equation (3): $\mu\, mg = \frac{mv^2}{r}$

$v^2 = rg\,\mu$

The maximum speed with which no skidding occurs is given by

$\boxed{v_{max} = \sqrt{\mu rg}}$

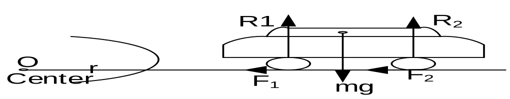
For no skidding

$\mu \geq \frac{v^2}{rg}$ Or $v^2 \leq \mu rg$

Example

1. A car of mass 1000kg goes round a bend of radius 100m at a speed of 50kmh^{-1} without skidding. Determine the coefficient of friction between the tyres and the road surface

Solution



$(\uparrow)$: $R_1 + R_2 = mg$------------------- (1)

$(\rightarrow)$: $F_1 + F_2 = \frac{m\,v^2}{r}$

$\mu(R_1 + R_2) = \frac{m\,v^2}{r}$------------------[2]

Put equation (1) and equation 2: μ mg= $\frac{m\,v^2}{r}$

$\mu = \frac{v^2}{rg} = \frac{\left(\frac{50x1000}{3600}\right)^2}{100x9.81} = 0.1965$

B) Overturning/toppling of a car

Consider a car of mass m moving around a horizontal (flat)circular bend of radius r at speed v. let the height of the centre of gravity above the track be "h" and the distance between the wheels be "2a".

$(\uparrow)$: $R_1 + R_2 = mg ------ (1)$

$(\rightarrow)$: $F_1 + F_2 = \frac{m\,v^2}{r} ------(2)$

Taking moments about G

$F_1.h + F_2.h + R_1.a = R_2.a$

$(F_1 + F_2)h + R_1 a = R_2.a$ -------------- (3)

Put equation 2 into equation 3

$\frac{m\,v^2}{r}.h + R_1 a = R_2.a$

$\frac{m\,v^2}{r}.\frac{h}{a} = (R_2 - R_1)$---------------------[4]

Equation 1+Equation 4

$R_1 + R_2 + \frac{m\,v^2}{r}.\frac{h}{a} = (R_2 - R_1) + mg$

$2R_1 = mg - \frac{mv^2h}{ra}$

$R_1 = \frac{m}{2}\left(g - \frac{v^2h}{ra}\right)$----------------------- (5)

A car just topples or upsets when $R_1 = 0$

$\frac{m}{2}\left(g - \frac{v^2h}{ra}\right)$ =0

$g = \frac{v^2h}{ra}$

$v_{max} = \sqrt{\frac{rga}{h}}$

9.7: Motion of a car on a banked track

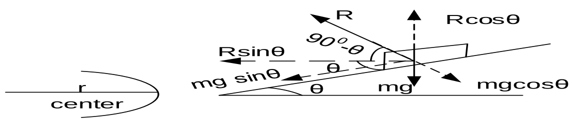
Definition :Banking a track is the building of a track round a corner with the outer edge raised above the inner one.

Advantages of banking

Banking ensures that only the horizontal component of normal reaction contributes towards the centripetal force.
Banking also enables the car to go round a bend at a higher speed for the same radius compared to a flat track.

A) SMOOTH TRACK

Consider a car of mass m negotiating a banked track at a speed v and radius of the bend is r.



$(\uparrow)$:$R\cos\theta = mg$ ------- (1)

$(\rightarrow)$:$R\sin\theta = \frac{m v^2}{r}$----(2)

(2) ÷ (1): $\frac{R\mathrm{Sin}\theta}{R\mathrm{Cos}\theta} = \frac{m v^2}{rmg}$

$\tan\theta = \frac{v^2}{rg}$

$v^2 = rg\tan\theta$

θ is the angle of banking and v is the designed speed of the banked track.

Example

1. A racing car of mass 1000kg moves around a banked track at a constant speed of 108kmh^{-1}, the radius of the track is 100m. Calculate the angle of banking and the total reaction at the tyres.

Solution

$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left[\frac{\left(\frac{108x1000}{3600}\right)^2}{100x9.81}\right]$ =42.5°

Resolving vertically:$R\cos\theta = mg$

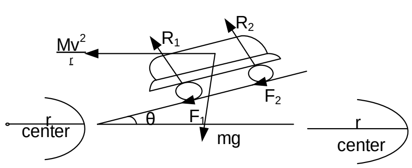
$R = \frac{1000x9.81}{\cos 42.5} = 13305N$

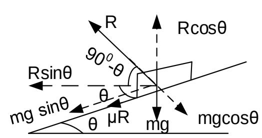
B) ROUGH TRACK

The frictional force must be there whose direction depends on the speed of the car.

(i) MAXIMUM SPEED/GREATEST SPEED

If the car is moving at speed v, greater than the designed speed v, the force Rsinθ is enough to provide the necessary centripetal force. The car will tend to slid outwards from the circular path, the frictional force would therefore oppose their tendency up to the maximum value .





$(\uparrow)$: $R\cos\theta = mg + \mu R\sin\theta$

$R(\cos\theta - \mu\sin\theta) = mg$ ------- (1)

$(\rightarrow)$: $R\sin\theta + \mu R\cos\theta = \frac{m v^2}{r}$

$R(\sin\theta + \mu\cos\theta) = \frac{m v^2}{r}$ ------ (2)

(2) ÷ (1): $\frac{R(\sin\theta + \mu\cos\theta)}{R(\cos\theta - \mu\sin\theta)} = \frac{m v^2}{rmg}$

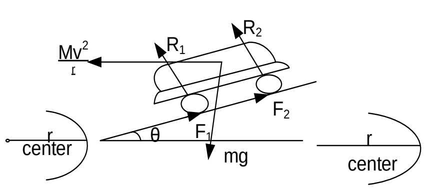
$\frac{(Sin\theta + \mu Cos\theta)}{(Cos\theta - \mu Sin\theta)} = \frac{v^2}{r}g$

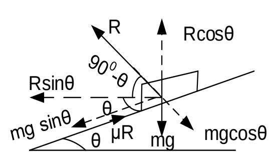
$v_{max}^2 = rg\frac{(Sin\theta + \mu Cos\theta)}{(Cos\theta - \mu Sin\theta)}$

Or divide the right hand side by cosθ

$v_{max}^2 = rg\left[\frac{(\tan\theta + \mu)}{(1 - \mu\tan\theta)}\right]$

(ii) MINIMUM SPEED/LEAST SPEED





$v_{min}^2 = rg\left[\frac{(\tan\theta - \mu)}{(1 + \mu\tan\theta)}\right]$

Examples

1. A car travels round a bend which is banked at 22^{0}. If the radius of the curve is 62.5m and the coefficient of friction between the road surface and tyres of the car is 0.3, calculate the maximum and minimum speed at which the car can negotiate the bend without skidding.

Solution

$$v_{max}^2 = rg\left[\frac{(tan\theta+\mu)}{(1-\mu tan\theta)}\right]$$

$$v_{max} = \left[62.5x9.81\left(\frac{tan22+0.3}{1-0.3tan22}\right)\right]^{\frac{1}{2}}$$ =**22.15ms^{-1}**

$$v_{min}^2 = rg\left[\frac{(tan\theta-\mu)}{(1+\mu tan\theta)}\right]$$

$$v_{min} = \left[62.5x9.81\left(\frac{tan22-0.3}{1+0.3tan22}\right)\right]^{\frac{1}{2}}$$ =**7.54ms^{-1}**

EXERCISE:29

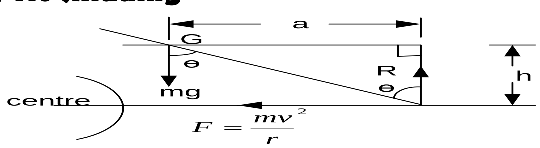
1. A racing car of mass 2 tonnes is moving at a speed of 5ms^{-1} round a circular path. If the radius of the track is 100m. calculate;
   i) Angle of inclination of the track to the horizontal if the car does not tend to side slip
   ii) The reaction to the wheel if it's assumed to be normal to the track. **An[1.5°, 19606.7N]**

.

9.8: Motion of a cyclist round a bend

A cyclist must bend towards the centre while travelling round the bend to avoid toppling. When the cyclist bends, the weight creates a couple which opposes the turning effect of the centrifugal forces.
Consider the total mass of the cyclist and his bike to be m round the circle of radius r at a speed v.

A) No skidding



(↑): $R = mg$ ---------- (1)

(→): $\mu R = \frac{m\,v^2}{r}$-------------------- (2)

Put 1 into 2: $\mu mg = \frac{m\,v^2}{r}$

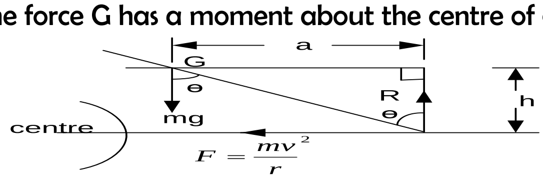
$$\boldsymbol{v^2 = \mu rg}$$

V is the max speed at which a cyclist negotiates a bend of radius r without skidding

For no skidding : $v^2 \leq \mu rg$

B) No toppling/ No over turning

The force G has a moment about the centre of gravity G(F.h) which tends to turn the rider out.



(↑) $R = mg$

Taking moment about G: $\frac{m\,v^2}{r}.h = R.a$

$$\frac{a}{h} = \frac{\frac{m\,v^2}{r}}{R}$$

But tan$\theta = \frac{a}{h}$

$$tan\theta = \frac{\frac{m\,v^2}{r}}{mg}$$

$$\boxed{v^2 = rgtan\theta}$$

v is the speed at which a cyclist can negotiate a corner without toppling

For no toppling $v^2 \leq rgtan\theta$

Why it is necessary for a bicycle rider moving round a circular path to lean towards a center of the path

When a rider moves round a circular path, the frictional force provides the centripetal force. The frictional force has a moment about the centre of gravity of the rider, the rider therefore tends to fall off from the centre of the path if this moment is not counter balanced. The rider therefore leans toward the center of the path so that his reaction provides a moment about the center of gravity , which counter balances the moment due to friction.

UNEB 2020 No3

a) (i) Define **centripetal acceleration** (01mark)

(ii) Show that the force F on a body of mass M moving in a circle of radius with constant speed V is given by $F = \frac{MV^2}{r}$ (05marks)

(iii) Derive the condition for a car to move round a banked circular track without slipping. (04marks)

UNEB 2019 No3

a) Define the following terms as applied ot circular motion

(i) Centripetal acceleration (01mark)

(ii) Period (01mark)

b) (i) Explain why a cyclist bends inwards while going around a curved path (03marks)

ii) Show that if θ is the angle of inclination of the cyclist to the vertical and μ is the coefficient of limititng friction between the ground and the bicycle tyres, then for safe riding $tan\theta \leq \mu$ (04marks)

(iii) A body of mass 1.5 kg moves once round a circular path to cover 44.0cm in 5.0 s. Calculate the centripetal force acting on the body. **An(0.166N)** (03marks)

UNEB 2014No1

(b) (i) Define angular velocity. (01mark)

(ii) satellite is revolving around the earth in a circular orbit at an altitude of $6\ x10^5m$ where the acceleration due to gravity is $9.4ms^{-2}$. Assuming that the earth is spherical, calculate the period of the satellite. **An[$5.42 \times 10^3 s$]** (03marks)

UNEB2013No3

(b) Show that the centripetal acceleration of an object moving with constant speed, v, in a circle of radius, r, is $\frac{v^2}{r}$ (04marks)

(c) A car of mass 1000kg moves round a banked track at a constant speed of 108kmh^{-1}. Assuming the total reaction at the wheels is normal to the track, and the radius of curvature of the track is 100m, calculate the;

(i) Angle of inclination of the track to the horizontal. **An[42.5°]** (04marks)

(ii) Reaction at the wheels **An[13305N]** (02marks)

UNEB 2007 No1

a) Explain why the maximum speed of a car on a banked road is higher than that on an unbanked road.

b) A small bob of mass 0.20kg is suspended by an inextensible string of length 0.8m. The bob is then rotated in a horizontal circle of radius 0.4m. find the

i) linear speed of the bob (3mk)

ii) tension in the string (2mk)

UNEB 2002 No1

d) The period of oscillation of a conical pendulum is 2.0s. if the string makes an angel 60° to the vertical at the point of suspension, calculate the

i) Vertical height of the point of suspension above the circle (3mk)

ii) Length of the string (1mk)

i) Velocity of the mass attached to the string (3mk)

An[0.995m, 1.99m, 5.41ms^{-1}]

CHAPTER 10: GRAVITATION

Gravitation deals with motion of planets in a gravitational field.

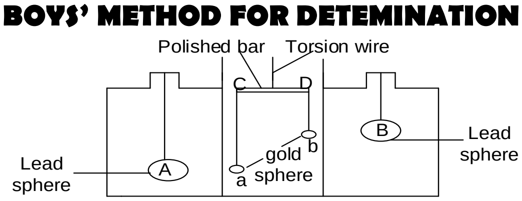
10.1: NEWTON'S LAW OF GRAVITATION

It states that: the force of attraction between two bodies in the universe is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

Exercise: 30

1. Calculate the gravitational attraction of two cars 5m part if the masses of the cars are 1000kg and 1200kg. **An**$(3.2x10^{-6}N)$
2. Calculate the force between the sun and Jupiter if the mass of the sun is 2.0$x10^{30}kg$, mass of Jupiter is $1.89x10^{27}kg$ and radius of Jupiter's orbit is $7.73x10^{11}m$. **An**$(4.22x10^{23}N)$
3. Calculate the force of attraction between two masses, one of 5kg and one of 8kg whose centres are 10cm apart. **An**$(2.7x10^{-7}N)$
4. Calculate the gravitational force of attraction between two 10kg particles which are 5cm apart. **An**$(2.7x10^{-6}N)$
5. Two small spheres of mass 4kg and mkg are placed 80cm apart. If the gravitational force is zero at a point 20cm from the 4kg mass along the line between the two masses, calculate the value of m. **An**$(36kg)$
6. A binary star consists of two dense spherical masses of 1.0$x10^{30}kg$ and 2.0$x10^{30}kg$ whose centres are 1.0$x10^{7}km$ apart and rotate together with and aangular velocity ω about an axis which intersects a line joining their centres. Find the value of ω and state two assumptions made. **An**$(1.4x10^{-5}rads^{-1})$
7. The earth is $6.0x10^{24}kg$ and that of the moon is $7.4x10^{22}kg$. If the distances between their centres is $3.8x10^{8}m$, calculate at what point on the line joining their centres is no gravitational force. **An**$(3.4x10^{8}m\ from\ earth)$
8. Two particles of mass 0.20kg and 0.30kg are placed 0.15m apart. A third particle of mass 0.05kg is placed between them on a line joining the first two particles. Calculate;
   (i) Gravitational force acting on the third particle placed 0.050m from 0.30kg mass
   (ii) Where along the line it should be placed for no gravitational force to be exerted on it. **An**$(3.35x10^{-10}N, 0.067M)$

10.3: BOYS' METHOD FOR DETEMINATION OF UNIVERSAL CONSTANT OF GRAVITATION, G



- Two identical gold sphere a and b of mass m are suspended from the ends of a highly polished bar CD of length l
- Two large spheres A and B each of mass M are brought in position near a and b respectively.
- The distance d between a and A or b and B is measured and recorded
- The deflection θ, of bar CD is measured by lamp and scale method.

Torque of couple on CD = $\frac{G\,m\,M}{d^2}$x l

$$\frac{G\,m\,Ml}{d^2} = k\theta$$

Where k is torsional of wire per unit radian of twist

$$G = \frac{k\theta d^2}{G\,m\,Ml}$$

10.4: GAVITATION FIELD STRENGTH/INTENSITY, g^1

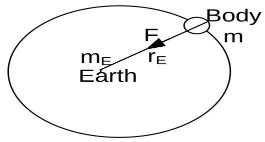
Gravitation field strength , g^1 at a point is the force on a unit mass placed at tha point.

The units are $N\,kg^{-1}$

Consider a body of mass m placed at a distance r from the centre of a planet of mass M, its Gravitation field strength , g^1 is given by

$$\frac{G\,Mm}{r^2} = mg^1 \qquad \boxed{GM = g^1\ r^2}$$

RELATION BETWEEN G AND g



Consider a body of mass m placed on the earth's surface of radius r_E where the acceleration due to gravity is g

$$\frac{G M_E m}{{r_E}^2} = mg$$

$$\boxed{G\, m_E = g\, {r_E}^2}$$

Where r_E is the radius of earth where $r_E = 6.4\text{x}10^6$m

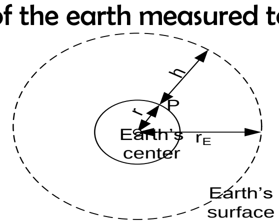
10.5: VARIATION OF GRAVITATIONAL FIELD STRENGTH

(i) Variation of field strength with height above the earth's surface

An object of mass m placed at a height h, above the surface of the earth where acceleration due to gravity at that height is g^1

(ii) Variation of field strength with depth below the earth surface

Consider the earth to be a uniform sphere of uniform density. Suppose a body is at a point h meters from the surface of the earth measured towards the centre of the earth.



When the object is on the surface of the earth .

$$mg = \frac{Gm_E m}{r_E^2}$$

$$M_E = \frac{{r_E}^2 g}{G} \text{ ------------------ (1)}$$

at P: $\quad {m_E}^1 g^1 = \frac{G\, {m_E}^1\, m}{r^2}$

$$m = \frac{r^2 g^1}{G} \text{ ---------------------------- (2)}$$

Where ${m_E}^1$ is the effective mass of that part of the earth which has a radius of r

Equation 2 divided by 1: $\frac{m}{M_E} = \dfrac{\frac{r^2 g^1}{G}}{\frac{{r_E}^2 g}{G}}$

$$\frac{m}{M_E} = \frac{r^2 g^1}{r_E^2 g} \text{ ------------------------- (3)}$$

For masses of uniform spheres are proportional to the cube of their radii

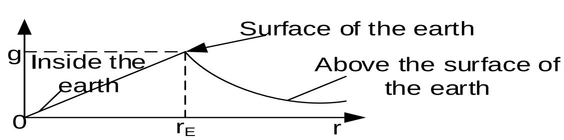
i.e. $m \propto r^3$ and $M_E \propto r_E^3$

$$\frac{r^3}{r_E^3} = \frac{r^2 g^1}{r_E^2 g}$$

$$g^1 = g\frac{r}{r_E}$$

$\therefore g^1 \propto r$ for a point inside the earth

(iii) Graph of variation of gravitational field strength from the centre of the earth



For points above the earth, the gravitational force obeys the inverse square law while for points inside the earth, g is proportional to the distance from the centre.

Examples

1. Calculate the gravitational field strength at a point above the earth surface which is 0.50m times the radius of the earth

Solution

$$\frac{g^1}{g} = \frac{{r_E}^2}{r^2}$$

$$g^1 = \frac{9.81x(r_E)^2}{(r_E + 0.5r_E)^2} = 4.36ms^{-2}$$

2. A body has a weight of 10N on the earth. What will its weight be on the moon if the ratio of the moon's mass to the earth's mass is 0.012 and the ratio of the moon's radius to that of the earth is 0.27?

Solution

moon's surface : $W_m = \frac{Gm_m\, m}{{r_m}^2}$----- (1)

earth's surface : $W_E = \frac{Gm_E m}{{r_E}^2}$

$$10 = \frac{Gm_E m}{{r_E}^2} \text{ ---------- (2)}$$

(2) $\div$ (1): $\frac{W_E}{10} = \frac{m_m}{m_E} x \left(\frac{r_E}{r_m}\right)^2 x10$

But $\frac{m_m}{m_e} = 0.012$ and $\frac{r_m}{r_e} = 0.27$

$$W_m = 0.012x\left(\frac{1}{0.27}\right)^2 x10 = 1.65N$$

EXERCISE:31

1. At what distance from the earth surface will the acceleration be $\frac{1}{8}$ of its value at the earth surface **An($1.18x10^7m$)**
2. Calculate the value of the gravitational intensity at a point
   (i) 8000m above sea level
   (ii) 8000m below sea level

   Take gravitational intensity at the surface of the earth as $10N\ kg^{-1}$ **An($9.975ms^{-2}, 9.988ms^{-2}$)**
3. The diameter of a black hole with the same mass as the earth is about 1.0cm. Calculate ;
   (i) The distance from the surface of the black hole where the gravitational intensity would be the same as that at the earth's surface.
   (ii) The gravitational intensity 1m from the centre of the black hole.

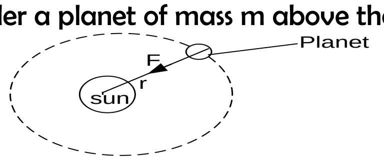
   Assume the laws of Physics are still obeyed near black holes. **An($6.33x10^6m$, $4x10^4ms^{-2}$)**
4. Mars has a radius of about 0.5 times tha of the eath and has a mass of approximately 0.1 of the earth. Find the gravitational field at the surface of the mars. **An($4ms^{-2}$)**

(iv) Variation of acceleration due to gravity with location on the surface of the earth

a) The earth is elliptical with the equatorial radius slightly greater than the polar radius. At the equator, the body is less attracted towards the earth than at the poles, acceleration due to gravity is greater at the poles than the equator
b) The earth rotates about its polar axis, weight of the body at the equator has to provide some centripetal force mω^2r where r is the equatorial radius, acceleration due to gravity is greater at the poles than the equator

10.7: VERIFICATION OF KEPLER'S 3RD LAW

Consider a planet of mass m above the sun of m_s. If the distance separating the planet and the sun is r.



centripetal force should be provided by the gravitational force of attraction

m r $\omega^2 = \frac{G\ mm_s}{r^2}$ but $\omega = \frac{2\pi}{T}$

$$m\,r\left(\frac{2\pi}{T}\right)^2 = \frac{G\ mm_s}{r^2}$$

$$T^2 = \left(\frac{4\pi^2}{Gm_s}\right)r^3$$

Since $\frac{4\pi^2}{Gm_s}$ is a constant $T^2 \alpha\ r^3$

Example

1. The average orbital radii about the sun of the earth and mars are $1.5x10^{11}m$ and $2.3x10^{11}m$ respectively. How many earth years does it take mars to complete its orbit

Solution

Earth takes 1 year to orbit around the sun

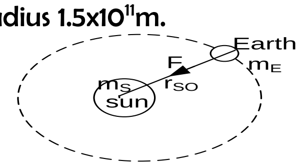
$$T^2 \alpha\ r^3$$
$$T^2 = k\ (2.3x10^{11})^3 \ldots\ldots\ldots.(i)$$
$$1^2 = k\ (1.5x10^{11})^3 \ldots\ldots\ldots.(ii)$$

$(i) \div (ii)$: $\frac{T^2}{1^2} = \frac{k(2.3x10^{11})^3}{k\ (1.5x10^{11})^3}$

$$T^2 = \left(\frac{23}{15}\right)^3 = 3.6$$
$$T = 1.9\ years$$

10.8: MASS OF THE SUN

The mass of the sun can be estimated by considering the motion of the earth round the sun in an orbit of radius 1.5x10[11]m.



Force of attraction= Centripetal force

$\frac{G\ M_E M_S}{r_{so}^2} = m_E\ \omega^2 r_{so}$

$m_s = \frac{\omega^2 r_{so}^3}{G}$ But $\omega = \frac{2\pi}{T}$

$$m_s = \frac{4\pi^2 r_{so}^3}{GT^2}$$

r_{so} is radius of the orbit of the earth around the sun

$r_{so} = 1.5x10^{11}$m

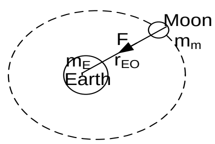
T =1yr≈365days =365x24x60x60s

r_{so} = $1.5x10^{11}$m

$$m_s = \frac{4x\left(\frac{22}{7}\right)^2 x\left(1.5x10^{11}\right)^3}{6.67x10^{11}x(365x24x60x60)^2} = 2.0x10^{30}kg$$

10.9: MASS OF THE EARTH

The mass of the earth can be estimated by considerably the motion of the moon round the earth in an orbit of radius $4x10^8$m



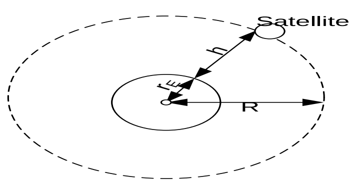
Force of attraction= Centripetal force

$$\frac{G\,M_E M_m}{r_{EO}^2} = m_m\,\omega^2 r_{EO}$$

$$m_E = \frac{4x\left(\frac{22}{7}\right)^2 x\left(4x10^8\right)^3}{6.67x10^{11}x(30x24x60x60)^2} = 5.6x10^{24}kg$$

10:10: PERIOD OF A SATELLITE

Consider a satellite of mass m moving in a circular orbit of radius h above the earth surface.



Attractive force=Centripetal force:

$$m\omega^2 R = \frac{Gm_E m}{R^2}$$

$$\omega = \frac{2\pi}{T}$$

$$\frac{m\,4\pi^2 R}{T^2} = \frac{Gm_E m}{R^2}$$

$$\boxed{T^2 = \frac{4\pi^2 R^3}{Gm_E}}$$

OR Where $R = r_E + h$

But also $Gm_E = g{r_E}^2$

$$\boxed{T^2 = \frac{4\pi^2 R^3}{g{r_E}^2}}$$

Examples

1. If the moon moves round the earth in a circular orbit of radius =$4.0x10^8$m and takes exactly 27.3 days to go round once, calculate the value of acceleration due to gravity g at the earth's surface. (04marks)

$m\omega^2 R = \frac{Gm_E m}{R^2}$ but $\omega = \frac{2\pi}{T}$

$$\frac{m\,4\pi^2 R}{T^2} = \frac{Gm_E m}{R^2}$$

But $G\,m_E = g\,{r_E}^2$

$$g = \frac{4\pi R^3}{T^2{r_E}^2}$$

$$g = \frac{4x\left(\frac{22}{7}\right)^2 x\left(4.0x10^8\right)^3}{(27.3x24x60x60)^2 x(6.4x10^6)^2} = 11.09ms^{-2}$$

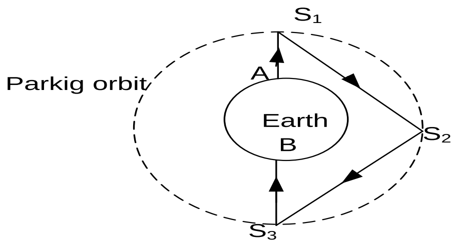
2. An artificial satellite move round the earth in a circular orbit in the plane of the equator at height 30,000km above the earth's surface (mass of earth =$6.0x10^{24}$kg, radius of the earth= $6.4x10^6$m,)
   i) Calculate its speed
   ii) What is the time between successive appearances over a point on the equator
   iii) What will be the additional distance of the satellite if it was to appear stationery

Solution

10.11: GEOSTATIONARY/SYNCHRONOUS ORBIT

These are communication satellites with orbital period of 24hrs and stays at the same point above the earth surface provided it is above the equator and its moving in the same direction as the earth is rotating.

HOW COMMUNICATION IS DONE USING A SATELLITE



- A set of three satellites are launched into geostationary or parking orbit
- Radio signals from A are transmitted to a geosynchronous satellite 1.
- These are re-transmitted from 1 to geosynchronous satellite 2.
- Then to geosynchronous satellite 3 which transmits to B

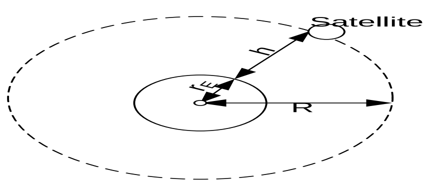
10.12: PARKING ORBIT

It's a path in space followed by a satellite which appears stationary when viewed from the earth surface.

Example

A communication satellite orbits the earth in synchronous orbits. Calculate the height of communication satellite above the earth.

Solution



Centripetal force= Attractive force:

But $R = r_E + h$

$$h = 4.22x10^7 - 6.4x10^6 = 3.58\,x10^7 m$$

10:13: GRAVITATIONAL POTENTIAL [U]

Gravitational potential at a point in the gravitational field is defined as the work done to move a unit mass from infinity to that point. Ie $U = \frac{w}{m}$

Examples

1. Assuming the earth is a uniform shere of radius 6.4x10^{6}m and its mass is 6.0x10^{24} kg, calculate
   (i) The gravitational potential at the earth surface
   (ii) The gravitational potential at a point 6.0x10^{5}m above the earth surface
   (iii) the work done in taking a 5kg mass from the earth's surface to a point 6.0x10^5 m above it
   (iv) the work done in taking a 5kg mass from the earth's surface to a point where the earth's gravitational effect is negligible.

Solution

(i) $U = \frac{-GM}{R}$

$$U_1 = -\frac{6.67x10^{-11}x6.0x10^{24}}{6.4x10^6} = -6.28x10^7 Jkg^{-1}$$

(ii) $U = \frac{-GM}{R}$

$R = R_E + h = (6.4x10^6 + 0.6x10^6)$ =**7.0x10^6**

$$U_2 = -\frac{6.67x10^{-11}x6.0x10^{24}}{7.0x10^6} = -5.74x10^7 Jkg^{-1}$$

$$W = (U_2 - U_1)xm$$
$$W = (-5.74x10^7 - -6.28x10^7)x5$$
$$W = 2.7x10^7 J$$

(iii) $W = (0 - U_1)xm$

$$W = (0 - -6.28x10^7)x5$$
$$W = 3.14x10^8 J$$

2. A body of mass 15kg is moved from the earth's surface to a point 2.8x10^{6}m above the earth. If the radius of the earth is 6.4x10^{6}m and its mass is 6.0x10^{24} kg, calculate the work done in taking the body to that point

Solution

10.14: ESCAPE VELOCITY

This is the minimum velocity with which a body is projected from the surface of the earth so that it escapes from the earth's gravitational pull.

10.16: EFFECT OF FRICTION ON A SATELLITE

- If a satellite is located within the earth atmosphere as it moves in its orbit, the atmospheric gasses offer frictional resistance to its motion. The satellite thus would be expected to do work to overcome this resistance and is so doing, it falls to an orbit of lower radius.
- The decrease in the radius causes the total energy $\left(\frac{-Gm_E\,m}{2R}\right)$ to decrease while the kinetic energy of the satellite $\left(\frac{Gm_E\,m}{2R}\right)$ increases resulting into an increase in the speed of the satellite in its new orbit. Because of the increase of the speed the satellite becomes hotter and it may burnout.

Examples

1. A satellite of mass 100kg is in a circular orbit at a height $3.59x10^7$m above the earth surface
   i) Calculate the kinetic energy, potential energy and the mechanical energy of the satellite in this orbit
   ii) State what happens when the mechanical energy of the satellite is reduced

Solution

i) $K.E = \frac{Gm_E\,m}{2R}$

$R = r_e + h$

K.E. $= \frac{6.67x10^{-11}x6x10^{24}x100}{2x(6.4x10^6+3.59x10^7)}$

K.E. $= 4.7.5x10^8$J

$P.E. = -\frac{Gm_E\,m}{R}$

$R = r_e + h$

J

$P.E. = -\frac{6.67x10^{-11}x6x10^{24}x100}{(6.4x10^6+3.59x10^7)}$

$P.E. = -9.4992\ x10^8$J

$M.E = P.E + K.E$

$= -9.4992\ x10^8 + 4.75x10^8$

M.E $= -4.75x10^8$J

(ii)

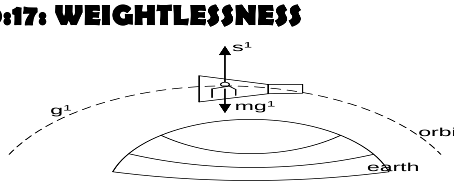
- ✓ Frictional force increases
- ✓ Satellite falls to an orbit of small radius
- ✓ PE reduces
- ✓ K.E increases
- ✓ Satellite becomes hot and may burn

EXERCISE: 32

2. The gravitational potential difference between two points is $3.0x10^3Jkg^{-1}$. Calculate the work done in moving a mass of 4.0kg between the two points. **An 1.2x10⁴J**]
3. The moon has mass $7.7x10^{22}kg$ and radius $1.7x10^6m$. Calculate;
   (i) The gravitational potential att its surface
   (ii) The work needed to completely remove a $1.5x10^3kg$ space craft from its surface into outer space. Neglect the effect of the earth, planet , sun, e.t.c **An [3.0x10⁶** Jkg^{-1}, $4.5x10^9J$]
3. A space station is in a stable circular orbit at a distance of 20,000km from the earth's centre. The radius of orbit of geostationary satellites is 42,000km. Find;
   (i) the orbital period of the space station
   (ii) Gravitational field strength at the space station. **An[**$7.8hours$, $0.98ms^{-2}$**]**
4. A preliminary stage of the space craft Apollo 11's journey to the moon was to place it in a parking orbit 189km above the earth's surface. Calculate
   (i) The gravitational intensity at this height
   (ii) The speed of the spacecraft
   (iii) The time to complete one orbit

An[9.21Nkg^{-1},7.8x10^{3}ms^{-1}, 5250 s]

10:17: WEIGHTLESSNESS



The sensation of weight is caused by the reaction of the floor on the person. In orbit an astronaut and the floor have the same acceleration as acceleration due to gravity. The floor therefore exerts no supporting force on the astronaut (zero reaction)
The astronaut therefore experiences a sensation of **weightless**.

Definition

Weightlessness is the condition of a zero reaction and a body moves with the same acceleration as acceleration due to the gravity.

UNEB 2017No2

(a) State **Kepler's laws** of planetary motion (03marks)
(b) Use Newton's law of gravitation to derive the dimension of the universal gravitational constant.(03marks)
(c) A satellite is revolving at a height h above the surface of the earth with a period , T
 (i) Show that the acceleration due to gravity g on the earth's surface is given by $g = \frac{4\pi^2(r_e+h)^3}{T^2r_e^2}$ where r_e is the radius of the earth (06marks)
 (ii) What is meant by **parking orbit** (02mark)
(d) A satellite revolves in a circular orbit at a height of $600km$ above the earth's surface. Find the
 (i) Speed of the satellite **An** $\mathbf{7.5764x10^3ms^{-1}}$ **or An** $\mathbf{7.542x10^3ms^{-1}}$ (03marks)
 (ii) Periodic time of the satellite **An** $\mathbf{5805.2s}$ or **An** $\mathbf{5802.2s}$ (03marks)

UNEB 2015No3

(a) State **Kepler's laws** of planetary motion (03marks)
(b) (i) What is a parking orbit (01mark)
 (ii) Derive an expression for the period, T of a satellite in a circular orbit of radius r, above the earth in terms of mass of the earth m, gravitational constant G and r (03marks)
(c) (i) A satellite of mas 200kg is launched in a circular orbit at a height of $3.59x10^7m$ above the earth's surface. Find the mechanical energy of the satellite **An** $\mathbf{-9.41x10^8\,J}$ (03marks)
 (ii) Explain what will happen to the satellite if the mechanical energy was reduced
(d) Describe a laboratory ,method of determining the universal gravitational constant, G (06marks)

UNEB 2004 No2

a) Explain and sketch the variation of acceleration due to gravity with distance from the centre of the earth. (06marks)

UNEB 2000 No 4

a) State Kepler's laws of gravitation (03marks)
b) I)Show that the period of a satellite in a circular orbit of radius r about the earth is given by

$$T = \left(\frac{4\,\pi^2}{G\,M_E}\right)^{\frac{1}{2}} r^{\frac{3}{2}}$$

Where the symbols have usual meanings (05marks)

ii)Explain briefly how world wide, radius or television communication can be achieved with the help of satellites (04marks)

c) A satellite of mass 100kg in a circular orbit at a height of $3.59x10^7$m above the earth's surface
 (i) Find the mechanical energy (04marks)
 (ii) Explain what would happened if the mechanical energy was decreased (04marks)

CHAPTER 11: SIMPLE HARMONIC MOTION (S.H.M)

Definition

This is the periodic motion of a body whose acceleration is directly proportional to the displacement from a fixed point and is directed towards the fixed point.

$$a \ \alpha - x$$
$$a = -\omega^2 x$$

The negative signs means the acceleration and the displacement are always in opposite direction.

Characteristics of SHM

(1) It's a periodic motion (to and fro motion)
(2) Mechanical energy is always conserved
(3) The acceleration is directed towards a fixed point
(4) Acceleration is directly proportional to its displacement

Practical examples of s.h.m

- Pendulum clocks
- Pistons in a petrol engine
- Strings in music instruments
- Motor vehicle suspension springs
- Balance wheels of a watch

a) Velocity in terms of displacement

Velocity of a body executing S.H.M can be expressed as a function of displacement x. this is obtained from the acceleration

$a = -\omega^2 x$

$a = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt}$

but $\frac{dx}{dt} = v$

$a = v \cdot \frac{dv}{dx}$

$v \cdot \frac{dv}{dx} = -\omega^2 x$

$vdv = -\omega^2 xdx$

integrating both sides

$\int v\, dv = -\omega^2 \int x\, dx$

$\frac{v^2}{2} = -\frac{\omega^2 x^2}{2} + C$ ....[1]

Where C is a constant of integration

When t = 0 v=0 and $x = r$(amplitude)

$\frac{0^2}{2} = -\frac{\omega^2 r^2}{2} + C$

$C = \frac{\omega^2 r^2}{2}$

Put into [1]: $\frac{v^2}{2} = -\frac{\omega^2 x^2}{2} + \frac{\omega^2 r^2}{2}$

$v^2 = \omega^2 r^2 - \omega^2 x^2$

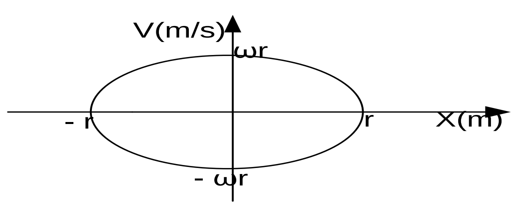
$\boxed{v^2 = \omega^2 (r^2 - x^2)}$

Velocity is maximum when x =0

$v^2 = \omega^2 r^2$

$\boxed{v_{max} = \omega r}$

A graph of velocity against displacement



From $v^2 = \omega^2 r^2 - \omega^2 x^2$

$v^2 + \omega^2 x^2 = \omega^2 r^2$

$\frac{v^2}{\omega^2 r^2} + \frac{x^2}{r^2} = 1$

This is an ellipse

Examples

1. A particles moves in a straight line with S.H.M. Find the time of one complete oscillation when
   i) The acceleration at a distance of 1.2m is 2.4ms^{-2}
   ii) The acceleration at a distance of 20cm is 3.2ms^{-2}

Solution

i) From $a = -\omega^2 x$

Negative is ignored

$2.4 = \omega^2 (1.2)$

$\omega^2 = \frac{2.4}{1.2}$

ω =1.4rads^{-1}

But $T = \frac{2\pi}{\omega}$

$T = \frac{2x\frac{22}{7}}{1.4} = 4.46s$

ii) $a = -\omega^2 x$

$3.2 = \omega^2 (0.2)$

ω =4rads^{-1}

$T = \frac{2\pi}{\omega} = \frac{2x\frac{22}{7}}{4}$ =1.57second

2. A Particle moving with S.H.M has velocities of $4ms^{-1}$ and $3ms^{-1}$ at distances of 3m and 4m respectively from equilibrium position. Find
   i) amplitude , ii) period , iii) frequency
   iv) velocity of the particle as it passes through equilibrium position

Solution

(i) $v = 4ms^{-1}, x = 3m$

Using $v^2 = \omega^2(r^2 - x^2)$

$4^2 = \omega^2(r^2 - 3^2)$

$16 = \omega^2(r^2 - 9)$------- (1)

Also $v = 3ms^{-1}, x = 4m$

$3^2 = \omega^2(r^2 - 4^2)$

$9 = \omega^2(r^2 - 16)$------- (2)

(1)÷ (2): $\frac{16}{9} = \frac{\omega^2(r^2 - 9)}{\omega^2(r^2 - 16)}$

$16(r^2 - 16) = 9(r^2 - 9)$

$r^2 = 25$

$r = 5m$; Amplitude =5m

(ii) period put $r = 5m$ into (1)

$4^2 = \omega^2(r^2 - 3^2)$

$16 = \omega^2(5^2 - 9)$

$\omega^2 = 1$

$\omega = 1$

But $T = \frac{2\pi}{\omega} = \frac{2x\frac{22}{7}}{1}$ =6.28second

(iii) frequency $= \frac{1}{T} = \frac{1}{6.28}$ =0.16Hz

(iv) velocity as it passes equilibrium position at equilibrium $x = 0$

$v^2 = \omega^2(r^2 - x^2)$

$v^2 = 1^2(5^2 - 0^2)$

$v = 5m/s$

Energy changes in s.h.m

- In S.H.M there's always an energy exchange. At maximum displacement , all the energy is elastic potential energy while at equilibrium point all the energy is kinetic energy

a) Kinetic energy

It's the energy possessed by a body due to its motion

K.E = ½ mv²= ½ m$\omega^2(r^2 - x^2)$

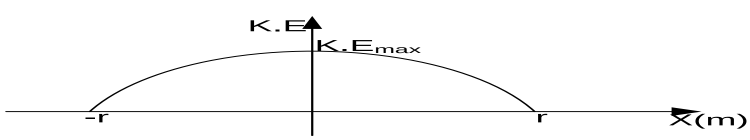
Note

i) The K.E is zero when the displacement x is equals to the amplitude

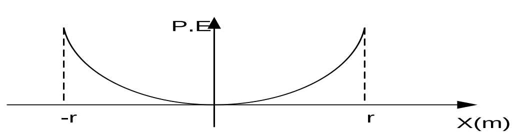
ii) K.E is maximum when the displacement x is zero

K.E_{max} = ½ m$\omega^2 r^2$

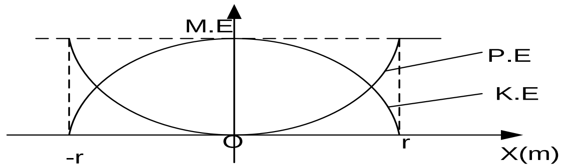
A graph of K.E against displacement



Graph of P.E against displacement



A graph of M.E against displacement



11.1: Mechanical oscillation

There are three types of oscillation i.e.

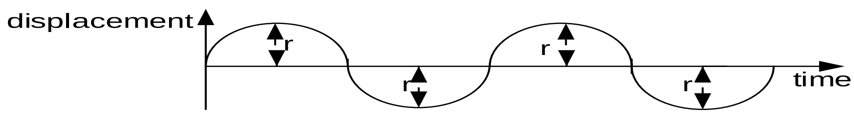
a) Free oscillation b) Damped oscillation c) Forced oscillation

a) Free oscillations

These are oscillations in which the oscillating systems does not do work against dissipative force such as air friction, and viscous drag and amplitude remains constant with time.

Eg a pendulum bob in a vacuum

Displacement- time graph



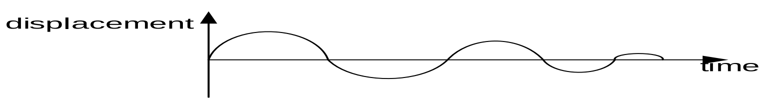
(a) Damped oscillations

These are oscillations in which the oscillating system loses energy to the surrounding due to dissipative forces and amplitude of these oscillations reduce with time

Types of damped oscillations

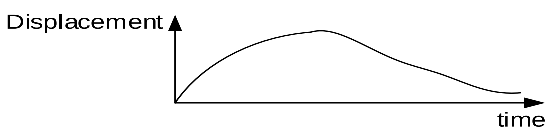
i) Under damped/slightly damped/lightly damped oscillations

Is when energy is lost and amplitude gradually decreases until oscillation dies away.



ii)Over damped/highly damped/heavily damped

Is when a system does not oscillate when displaced but takes a very long time to return to equilibrium position.

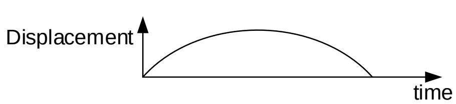


Example

- A horizontal spring with a mass on a rough surface

iii) Critically damped oscillations

Is when a system does not oscillate when displaced and returns to equilibrium position in a short time.



Example

- Shock absorber in a car

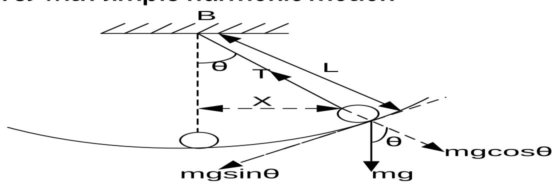
C) FORCED OSCILLATIONS

These are oscillations where the system is subjected to an external force and the system oscillates at the same frequency as the oscillating force.

Examples of S.H.M

11.2: SIMPLE PENDULUM

Consider a mass m suspended by a light inelastic string of length L from a fixed point B.
If the bob is given a small vertical displacement through an angle θ and released, we show that a bob moves with simple harmonic motion



Resolving tangentially: Restoring force = $-mgsin\theta$

By Newton's 2nd law: $ma = -mgsin\theta$

$ma = -mgsin\theta$.....................................1

If the displacement is small, then θ is very small.

$$Sin\theta \approx \theta \approx \frac{x}{l}$$

$$ma = -mg\theta$$

$$a = -g\frac{x}{l} = -\left(\frac{g}{l}\right)x$$

it is in the form $a = -\omega^2 x$ and hence performs S.H.M with period

$$\omega^2 = \frac{g}{l}$$

$$\boxed{\omega = \sqrt{\frac{g}{l}}}$$

But $= \frac{2\pi}{T}$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}}$$

Determination of acceleration due to gravity (g) using a simple pendulum

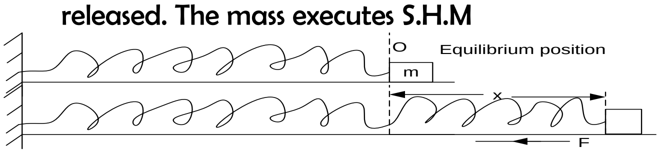
- Starting with a measured length L of the pendulum, the pendulum is clamped between 2 wood pieces from a retort stand.
- A bob is then given a small angular displacement from the vertical position and released.
- The time t for 20 oscillation is obtained, find period T and henceT^2
- Repeat the procedure for different values of length of the string.
- A graph of T^2 against L is then drawn and its slope S calculated.

Hence acceleration due to gravity is obtained from $g = \frac{4\pi^2}{s}$

11:2: MASS ON A SPRING

a) A horizontal spring attached to a mass

Consider a spring lying on a smooth horizontal surface in which one end of the spring is fixed and the other end attached to a particle of mass m. When the mass is slightly pulled a small distance x and the released. The mass executes S.H.M



By Hooke's law :$F = -kx$--------------------- (1)

By Newton's 2nd law: $ma = -kx ---- (2)$

$$\boxed{\omega = \sqrt{\frac{k}{m}}}$$

Example : UNEB 2011 No 4C

A horizontal spring of force constant 200 Nm^{-1} is fixed at one end and a mass of 2kg attached to the free end and resting on a smooth horizontal surface. The mass is pulled through a distance of 4.0cm and released. Calculate the;

i) Angular speed

ii) Maximum velocity attained by the vibrating body, acceleration when the body is half way towards the centre from its initial position.

Solution

i) From $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{2}} = 10$rads^{-1}

ii) $v_{max} = \omega r$

$v_{max} = 10x\frac{4}{100}$ =0.4ms^{-1}

Note: the small distance pulled and released becomes the amplitude

$$a = -\omega^2 x$$

where its half towards the centre

$x = \frac{r}{2}$

$x = \frac{4x10^{-2}}{2}$

$a = -\omega^2 x = 10^2 x \frac{4x10^{-2}}{2} = 2ms^{-2}$

Alternatively

$F = ma$

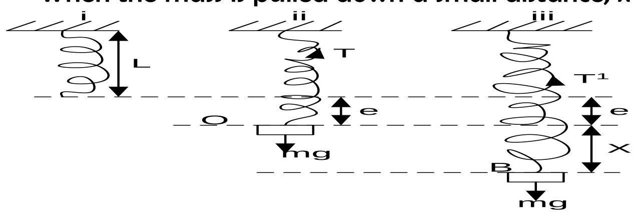
$F = kx$

$k\frac{r}{2} = ma$

$a = \frac{200x4x10^{-2}}{2x2} = 2ms^{-2}$

b) Oscillation of mass suspended on a helical spring

Consider a helical spring or elastic string suspended from a fixed point.
When a mass is attached to the spring, it stretches by length, e and comes to equilibrium positions O.
When the mass is pulled down a small distance, x and released the motion will be simple harmonic.



In position (ii) the mass is in equilibrium position

$T = mg$

And by hooke's law $T = ke$

$mg = ke$ -------------------------(1)

In position (iii) after displacement through x

But by Hooke's law $T^1 = k(e + x)$

By Newton's 2nd law: $mg - k(e + x) = ma$

But from equation 1 $mg = ke$

$ke - k(e + x) = ma$

$ke - ke - kx = ma$

$-kx = ma$

$a = -\frac{k}{m}x$ ------------------------------- [2]

Determination of acceleration due to gravity using a vertically loaded spring

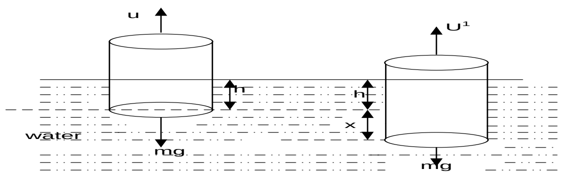
- Clamp a spring on a retort stand using pieces of wood
- Fix a horizontal pin to the free end of the spring to act as a pointer
- Place a vertical meter rule next to the pin and note its initial position
- Suspend a known mass, m at the free end of the spring, note and record the new position of the pointer
- Calculate the extension e produced
- Repeat the procedure above for several masses suspended in turns and tabulate.
- Plot a graph of **e** against **m**
- Find the slope, s of the graph

Hence acceleration due to gravity is determined from $g = ks$ where k is known spring constant

Alternatively

11.4: S.H.M OF A FLOATING CYLINDER

Consider a uniform cylindrical rod of length L and cross sectional area A and density, ρ floating vertically in a liquid of density, σ. When the rod is given a small downward displacement x and released, the rod executes S.H.M.



At equilibrium, $U = mg = Ah\delta g$ -------------- [1]

After a downward, restoring force $F = U^1 - mg$

$F = A(h+x)\delta g - Ah\delta g$ ---------- [2]

But $F = ma$ hence

$$Ah\delta g - A(h+x)\delta g = ma$$

$$-A\delta g\, x = ma$$

$$a = -\left(\frac{A\delta g}{m}\right)x$$

But $m = A\, l\, \rho$

$$a = -\left(\frac{A\delta g}{A\, l\, \rho}\right)x$$

$a = -\left(\frac{\delta g}{l\,\rho}\right)x$-------------------------- [3]

it is the form $a = -\omega^2 x$

$$\omega^2 = \frac{\delta g}{l\,\rho}$$

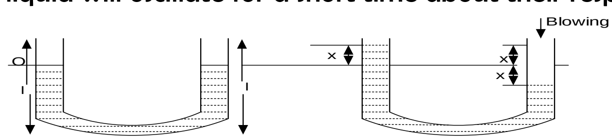
$\boxed{\omega = \sqrt{\frac{\delta g}{l\,\rho}}}$---------------------------- [4]

$$T = 2\pi\sqrt{\left(\frac{\rho l}{\delta g}\right)}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{\delta g}{l\,\rho}}$$

11.5: A LIQUID OSCILLATING IN A U-TUBE

Consider a column of liquid of density δ and total length l in a U-tube of uniform cross sectional area A. Suppose the level of the liquid on the right side is depressed by blowing gently down that side, the levels of liquid will oscillate for a short time about their respective or equilibrium positions O.



When the meniscus is at a distance, , from equilibrium position, a differential height of liquid of, $2x$, is produced

Excess pressure on liquid $= 2x\delta g$ from $[p = h\delta g]$

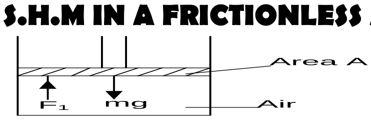
Force on liquid ,$F = 2x\delta gA$

Restoring force $F = -ma$ ----------------[1]

Newton's 2nd law : $ma = -2x\delta gA$

$a = -\left(\frac{2\delta gA}{m}\right)x$-----------------------------[2]

11.6: S.H.M IN A FRICTIONLESS AIR TIGHT PISTON



At Equilibrium : $F_1 = PA$

$PA = mg$ ------------------------------- [1]

When the piston is given a slight downward displaced x,

the restoring force $F_2 = P_2A - mg$

But by Newton's 2nd law : $ma = -[P_2A - mg]$

from Equation 1: $PA = mg$

$ma = -(P_2A - PA)$--------------------- [2]

Boyle's law. $[P_1V_1 = P_2V_2]$

$$P_2(v - Ax) = Pv$$

$$P_2 = \frac{Pv}{(v - Ax)}$$

$$ma = -\left(\frac{Pv}{(v - Ax}A - PA\right)$$

$$ma = -PA\left(\frac{Ax}{v - Ax}\right)$$

For small displacement, x: $v - Ax \approx v$

$$ma = -PA\left(\frac{Ax}{v}\right)$$

$$ma = -A\left(\frac{PAx}{v}\right)$$

$$a = -\left(\frac{PA^2}{m\,v}\right)x$$

it is in the form $a = -\omega^2 x$

$$\omega^2 = \frac{PA^2}{m\,v}.$$

$$\omega = \sqrt{\frac{PA^2}{m\,v}}$$

$$\omega = A\sqrt{\frac{P}{m\,v}}.$$

$$f = \frac{A}{2\pi}\sqrt{\frac{P}{m\,v}}$$

Example

A piston in a car engine performs S.H.M. The piston has a mass of 0.50kg and its amplitude of vibration is 45mm. the revolution counter in the car reads 750 revolutions per minute. Calculate the maximum force on the piston.

Solution

$r = 45mm = 45x10^{-3}m,\ m = 0.5kg$

$f = 750\,rev/min = \frac{750}{60} = 12.5rev/s$

But $a_{max} = \omega^2 r$

$\omega = 2\pi f$

$a_{max} = (2\pi f)^2 r$

$a_{max} = \left(2x\frac{22}{7}x12.5\right)^2 x12.5$

$a_{max} = 277.583ms^{-2}$

$F_{max} = ma_{max} = 0.5x277.583 = 138.792N$

UNEB 2020 No 3

(b) Describe how a helical sopring may be used to determine the acceleration due to gravity. [5mks]

(c) A particle moving with s.h.m has a speed of 8.0m/s and an acceleration of 12ms^{-2} when it is 3.0m from its equilibrium position. Find the;

(i) amplitude of motion [3mks]

(ii) maximum acceleration [2mks]

UNEB 2019 No 3

(c) Define **simple harmonic motion** [1mk]

(d) A body executes s.h.m with amplitude A and angular velocity ω

(i) Write down the equation for the velocity of the body at a displacement x from the mean Position [1mk]

(ii) Sketch a velocity- displacement graph for a body in (d)(i) for $\omega < 1$ [2mks]

(iii) If the body moves with amplitude 14.142cm, at what distance from the mean position will the kinetic energy be equal to potential energy? **An(10cm)** [3mks]

UNEB 2017 No 3

a) (i) Define **simple harmonic motion** [1mk]

(ii) Sketch a displacement-time graph for a body performing simple harmonic motion [1mk]

b) A uniform cylindrical rod of length 16cm and density $920kgm^{-3}$ float vertically in a liquid of density $1000kgm^{-3}$. The rod is drepressed through a distance of 7mm and then released.

i) Show that the rod executes simple harmonic motion [06mk]

ii) Find the frequency of the resultant oscillations **An(1.299Hz)** [04mk]

iii) Find the velocity of the rod when it is at a distance of 5mm above the equilibrium position **An($3.998x10^{-2}ms^{-1}$)** [03mk]

c) What is meant by potential energy [01mk]

d) Describe energy changes which occur when a

(i) Ball is thrown upwards in air [03mk]

(ii) Loud speaker is vibrating [01mk]

UNEB 2009 No 3

(a) What is meant by simple harmonic motion (01marks)

(b) A cylindrical vessel of cross-sectional area A, contains air of volume V, at a pressure P, trapped by frictionless air tight piston of mass M,. The piston is pushed down and released.

(i) If the piston oscillates with s.h.m, show that the frequency is given by $f = \frac{A}{2\pi}\sqrt{\frac{P}{m\,v}}$ (06marks)

(ii) Show that the expression for, f in b(i) is dimensionally correct (02marks)

(c) Particle executing s.h.m vibrates in a straight line, given that the speeds of the particle are 4m$s^{-1}$and 2ms^{-1} when the particle is 3cm and 6cm respectively from equilibrium. calculate the;

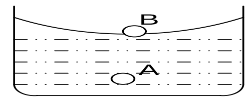
(i) amplitude of oscillation **An(6.7x10^{-2}m)** (03marks)

(ii) frequency of the particle **An(10.68Hz)** (03marks)

(d) Give two examples of oscillatory motions which execute s.h.m and state the assumptions made in each case

CHAPTER 12: SURFACE TENSION

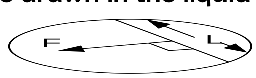
12.1.1: Molecular explanation for existence of surface tension



- Liquid molecules attract each other. In the bulk of the liquid the resultant force on any molecule such as A is zero.
- A surface molecule such as B is subjected to intermolecular forces of attraction below therefore potential energy of surface molecules exceeds that of the interior. Average separation of the surface molecules is greater than that of molecules in the interior. At any point on a liquid surface there is a net force away from that point and this makes the surface behave like an elastic skin in a state of tension. This accounts for surface tension.

Definition

Surface tension coefficient γ of a liquid is defined as the force per unit length acting at right angles to one side of an imaging line drawn in the liquid surface.

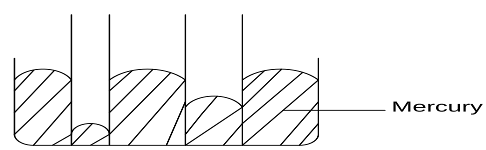


$\gamma = \frac{F}{L}$

12.2.0: CAPILLARITY

If the capillary tube is dipped inside mercury liquid is depressed below the outside level. This is because the cohesion of mercury is greater than the adhesion of mercury and glass.

The depression of the tube increases with decreases the diameter of the tube



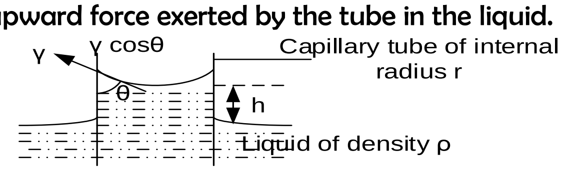
Definition

Capillarity: Is the rise or fall of a liquid in a capillary tube

12.2.1: Capillary rise

Around the boundary where the liquid surface meets the tube, surface tension forces exert a downward pull on the tube since they are not balanced by any other surface tension forces.

The tube therefore exerts an equal but upwards force on the liquid which forces it to rise. The liquid stops rising when the weight of the raised column acting downwards equals to vertical component of the upward force exerted by the tube in the liquid.



Force acting upwards $F = \gamma cos\ \theta\ x\ L$

But L=2πr

F = γ cos θx2πr ------------[1]

Weight W= mg =Vρg

W =Ahρg= πr^{2}hρg ----------------- [2]

At equilibrium: $W = F$

πr^{2}hρg= $\gamma\ cos\ \theta x 2\pi r$

h= $\frac{2\,\gamma\ cos\theta}{r\ \rho\ g}$

Examples

1. A clean glass capillary tube of internal diameter 0.04cm is held with its lower end dipped in water contained in a beaker and with 12cm of the tube above the surface of water.
   - i) To what height will water rise in the tube.
   - ii) What will happen if the tube is now depressed until only 4cm of its length is above the surface. (γ of water =7.0x10^{-2}Nm^{-1}, ρ of water =1000kgm^{-3})

Solution

i) Using $h = \frac{2\,\gamma\cos\theta}{r\,\rho\,g}$

But for a clean glass of water θ =0

$$h = \frac{2\ x\ 7x\ 10^{-2}\ cos0}{\left(\frac{0.04\ x10^{-2}}{2}\right) x\ 1000\ x9.81} = 0.071\text{m}$$

ii) If only 4cm of the tube is left above the water surface, this length is less than h in part (i) above so water must change its angle of contact so that it can fit the 4cm

$$h = \frac{2\,\gamma\cos\theta}{r\,\rho\,g}$$

$$4x10^2 = \frac{2\ x\ 7x\ 10^{-2}\ cos0}{\left(\frac{0.04\ x10^{-2}}{2}\right) x\ 1000\ x9.81}$$

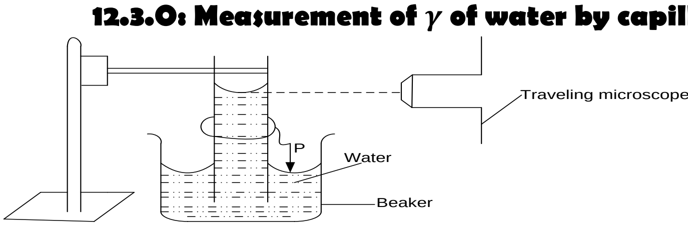
θ=55.9°

water forms a new surface with an angle of contact 56°

Exercise: 34

1. A liquid of density 1000kg$m^{-3}$and surface tension $7.26x10^{-2}Nm^{-1}$, dipped in it is a capillary tube with a bore radius of 0.5mm. If the angle of contact is 0° determine,
   - i) the height of the column of the liquid rise
   - ii) if the tube is pushed until its 2cm above the level of the liquid, explain in what happen **An[2.96x10^{-2}m, 47.5°]**
2. The two vertical arms of manometer containing water, have different internal radii of 10^{-3}m and 2x10^{-3}m respectively. Determine the difference in height of the two liquids levels when the arms are open to the atmosphere. (surface tension and density of water are $7.2x10^{-2}Nm^{-11}$and 10^3kgm^{-3} respectively) **An[7.14x10^{-3}m]**
3. Calculate the height to which the liquid rises in the capillary tube of diameter 0.4mm placed vertically inside
   - (i) A liquid of density $800kgm^{-3}$ and surface tension $5x10^{-2}Nm^{-1}$ and angle of contact 30°
   - **(ii)** Mercury of angle of contact 139° and surface tension 0.52Nm^{-1} **An[0.032m, 0.0294m]**

12.3.0: Measurement of γ of water by capillary tube method

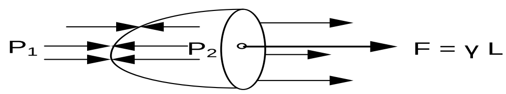


- A clean capillary tube is dipped in water as shown above and a wire P which is bent is tied along the capillary tube with a rubber band.
- When the tube is dipped into water, the wire P is adjusted so that its top just touches the surface of the water.
- A travelling microscope is focused on the water meniscus in the capillary tube and the reading noted, say h_1.
- The beaker is then removes and the travelling microscope is focused on the tip of the wire P and scale reading h_2 is noted.
- The height of the water rise, h is calculated from $h = /h_1 - h_2/$.
- The capillary tube is removed and its diameter and hence radius , r is determined by using a travelling microscope. The surface tension can be obtained from ;

$\gamma = \frac{h\,r\,\rho\,g}{2\cos\theta}$ for clean glass of water θ=0°

12.4.1: Pressure difference across an air bubble

Consider an air bubble of radius r which is spherical and formed in a liquid of surface tension γ. Therefore the total length of surface in contact with air is L such that surface tension force.



P_1 = External pressure on the bubble due to the liquid
P_2 = internal pressure of air in the bubble

For the bubble to maintain its shape the, internal pressure should be bigger than the external pressure.

At equilibrium; Force due to P_2 = force due to P_1 + surface tension force

$$AP_2 = AP_1 + \gamma L$$

$$\pi r^2 P_2 = \pi r^2 P_1 + 2\pi r\,\gamma$$

$$\pi r^2 (P_2 - P_1) = 2\pi r\,\gamma$$

$$\boxed{P_2 - P_1 = \frac{2\gamma}{r}}$$

OR Excess pressure $= \frac{2\gamma}{r}$

1. Calculate the total pressure within an air bubble of diameter 0.1mm formed at depth of 10cm below the water surface. (Atmospheric pressure is $1.013x10^5 Pa$ and surface tension of water is 0.0727Nm^{-1}).
An $1.039x10^5 Pa$

12.4.2: Excess pressure (pressure difference) for a soap bubble

A soap bubble has a diameter of 4mm. Calculate the pressure inside it, if the atmospheric pressure is 10^5Nm^{-2}, and that the surface tension of soap solution is $2.8x10^{-2} Nm^{-1}$

Solution

$P_2 - P_1 = \frac{4\gamma}{r}$

$P_2 = \frac{4\ x2.8x10^{-2}}{2\ x\ 10^{-3}} + 10^5 = 1.001x10^5 Pa$

12.5.0: FREE SURFACE ENERGY (σ)

It is defined as the work done in increasing area of the surface by $1m^2$ under isothermal conditions .
Units of σ are Jm^{-2} or Nm^{-1}

Examples

1. Calculate the work done against surface tension force on blowing a soap bubble of diameter 15mm , if the surface tension of the soap solution is 3.0x10^{-2}Nm^{-1}.
Solution

$\gamma = \frac{work\ done}{increase\ in\ S.A}$

Work done = γ x increase in surface area

$= \gamma x(2x4\pi r^2)$=3.0$x10^{-2}x$2x4$x\frac{22}{7}x\left(\frac{15x10^{-3}}{2}\right)^2$

Work done = 4.241$x10^{-5}$J

Increases in surface area is multiplied by 2 for both the upper and lower surface of a soap bubble.

2. Calculate the work done in breaking up a drop of water of radius 0.5cm in to tiny droplets of water each of radius 1mm assuming isothermal conditions, given that surface tension of water is $7x10^{-2}$ Nm^{-1}.
Solution

Radius of big drop, R= 0.5cm $= 5x10^{-3}m$ and
Radius of n tiny droplets, r= 1mm $= 1x10^{-3}m$
Volume of big drop= $\frac{4}{3}\pi R^3 = \frac{4}{3}\pi(5x10^{-3})^3$
Volume of n tiny droplets= $nx\frac{4}{3}\pi r^3 = nx\frac{4}{3}\pi(1x10^{-3})^3$

$$nx\frac{4}{3}\pi(1x10^{-3})^3 = \frac{4}{3}\pi(5x10^{-3})^3$$

$\boldsymbol{n} = 125\ droplets$

Work done = γ x increase in surface area= γx(4πr^2)
Big drop: Work done= $7x10^{-2}\ x4\pi(5x10^{-3})^2$
= 2.2$x10^{-5}$J

125 drop lets:
Work done= $125x7x10^{-2}\ x4\pi(1x10^{-3})^2$
= 1.1$x10^{-4}$J

Change in surface energy =1.1$x10^{-4} - 2.2x10^{-5}$
$= x1.09x10^{-4}$J

EXERCISE: 35

1. A spherical drop of mercury of radius 2mm falls to the ground and breaks into 10smaller drops of equal size. Calculate the amount of work that has to be done.
   (Surface tension of mercury = 4.72×10^{-1} Nm^{-1}) **An[2.74×10^{-5}J]**

COMBINED BUBBLES

CASE 1

A soap bubble x of radius r_1, and another bubble y of radius r_2, are brought together so that the combined bubble has a common interface of radius R. show that

$R = \frac{r_1 \; r_2}{r_2 - r_1}$

CASE 2

Two bubbles of a soap solution of radii r_1 and r_2 of surface tension γ and pressure P coalence under isothermal conditions to form one bubble. Find the expression for the radius of the bubble formed.

Solution

Let R be the radius of the new bubble

A_1 be the surface area of bubble with radius r_1

A_2 be the surface area of bubble with radius r_2

A be the surface area of bubble with radius R

Under isothermal conditions, work done in enlarging the surface area of a bubble is given by

$$2\gamma A = 2\gamma A_1 + 2\gamma A_2$$

$$2\gamma 4\pi R^2 = 2\gamma 4\pi\, r_1{}^2. + 2\gamma 4\pi\, r_2{}^2$$

$$R^2 = r_1{}^2 + r_2{}^2$$

$$R = \sqrt{r_1{}^2 + r_2{}^2}$$

UNEB 2017 Q.4

(a) A capillary tube is held in a vertical position with one end dipping in a liquid of surface tension γ and density ρ. If the liquid rises to a height, h derive an expression for h in terms of γ, ρ and radius r of the tube assuming the ange of contact is zero. (04mks)

(b) A mercury drop of diameter 2.0mm falls vertically and on hitting the ground, it splits into two drops each of radius 0.5mm. If the surface tension of the mercury is 0.52Nm^{-1} calculate the resulting change in surface energy **An** ($2.289x10^{-5}$J)
(05mks)

(c) State the effect of temperature on surface tension of a liquid. (01mk)

UNEB 2002 Q.4

a) Define the term surface tension in terms of surface energy (01mark)

b) i) Calculate the work done against surface tension is blowing a soap bubble of diameter 15mm, if the surface tension of the soap solution is $3.0 \times 10^{-2} Nm^{-1}$ **An** [4.24×10^{-5}J] (03marks)

ii) A soap bubble of a radius r_1 is attached to another bubble of radius r_2. If r_1 is less than r_2. Show that the radius of curvature of the common interface is $\frac{r_1 \; r_2}{r_2 - r_1}$ (05marks)

UNEB 2001 Q.3

a) Define surface tension and derive its dimension (3mk)

b) Explain using the molecular theory the occurrence of surface tension (4mk)

c) Describe an experiment to measure surface tension of a liquid by the capillary tube method (6mk)

d) i)Show that the excess pressure in a soap bubble is given by $P = \frac{4\gamma}{r}$

ii) Calculate the total pressure within a bubble of air of radius 0.1mm in water, if the bubble is formed 10cm below the water surface and surface tension of water is $7.27 \times 10^{-2} Nm^{-1}$. [Atmospheric pressure $= 1.01 \times 10^5$Pa] **An 1.03×10^5Pa**

SECTIONB: HEAT AND THERMO DYNAMIC

1.1: THERMOMETRIC PROPERTY

A thermometric property is a physical property which varies linearly and continuously with change in temperature.

QUALITIES OF A GOOD THERMOMETRIC PROPERTY

- It should vary linearly with change in temperature
- It should vary continuously with change in temperature
- It should be measurable over a wide range of temperature
- It should be sensitive to temperature changes
- Property should be accurately measurable with a single apparatus

1.2: FIXED POINT

This is temperature at which a substance changes states under specific conditions.

Note

Triple point of water is taken as a standard in thermometry instead of ice and steam point because triple point is a single temperature and a single pressure while ice point and steam point vary with pressure and some level of impurities.

(a): CENTIGRADE/ CELSIUS TEMPERATURE SCALE

Is a temperature scale which uses ice point (0°C) as it lower fixed point and steam point (100°C) as its upper fixed point

STEPS IN SETTING UP CELSIUS TEMPERATURE SCALE

- Choose a thermometric properly of substance and let it be X
- Measure the value of the property at ice point, steam point and let values be X_0, X_{100} respectively.
- Measure the value of the properly at unknown temperature θ and let it be X_θ
- Unknown temperature is determined from $\theta = \left(\frac{X_\theta - X_0}{X_{100} - X_0}\right) x100°\text{C}$

(b) KELVIN / THERMODYNAMIC TEMPERATURE SCALE

This is a temperature scale which uses triple point of water as a fixed point.

Kelvin is defined as $\frac{1}{273.16}$ of the thermodynamic temperature of the triple point of water

Examples

1) A resistance thermometer has a resistance of 21.42Ω at ice point, 29.10 Ω at steam point and 28.11 Ω at un known temperature θ. calculate θ on scale of this thermometer.

Solution

$$\theta = \left(\frac{R_\theta - R_0}{R_{100} - R_0}\right) x100°\text{C}$$

$$\theta = \left(\frac{28.11 - 21.42}{29.10 - 21.42}\right) x100°\text{C}$$

$$\theta = 87.11°\text{C}$$

2) Pressure recorded by constant volume thermometer at Kelvin temperature T is given by $4.8x10^4 Nm^{-2}$. Calculate T if the pressure at triple point of water is $4.2x10^4 Nm^{-2}$

Solution

$$T = \frac{P_T}{P_{tr}} x273.16K$$

$$T = \frac{4.8x10^4}{4.2x10^4} x273.16K = 312.18K$$

Determining temperature on a scale of one thermometer as read by another

1) The resistance, R_θ of a particular resistance thermometer at Celsius temperature θ as measured by a constant volume gas thermometer is. $R_\theta = 50 + 0.17\theta + 3x10^{-4}\,\theta^2$ Calculate the temperature as measured on a scale of a resistance thermometer which corresponds to a temperature of 60℃ at a gas thermometer.

Solution

$$R_\theta = 50 + 0.17\theta + 3x10^{-4}\,\theta^2$$
$$R_0 = 50 + 0.17x0 + 3x10^{-4}x\,0^2 = 50$$
$$R_{100} = 50 + 0.17x100 + 3x10^{-4}x\,100^2 = 70\Omega$$
$$R_{60} = 50 + 0.17x60 + 3x10^{-4}x\,60^2 = 61.28\Omega$$
$$\theta = \left(\frac{R_\theta - R_0}{R_{100} - R_0}\right)x100℃ = \left(\frac{61.28-50}{70-50}\right)x100℃$$
$$\theta = 56.4℃$$

2) The resistance of the wire as measured by gas thermometer varies with temperature θ according to the equation. $R_\theta = R_0\,(1 + 50\alpha\theta + 200\alpha\theta^2)$.Determine temperature on resistance thermometer that corresponds to 40°Con the gas scale

Solution

$\theta = 16.02℃$

Exercise: 37

1) The resistance of the element in a platinum resistance thermometer is 6.75 Ω at triple point of water and 7.166 Ω at room temperature. What is the temperature of the room on a scale of resistance thermometer?. state one assumption you have made. **An[290K]**

2) The resistance of the wire is measured at ice point, steam point and at un known temperature θ and the following values were obtained 2.00 Ω, 2.48 Ω, 2.60 Ω respectively. Determine θ **An**[125℃]

3) The resistance R of platinum wire at temperature θ°C as measured by a constant volume thermometer is given by; $R_\theta = R_0(1 + 8000\alpha\theta - \alpha\theta^2)$ where α is a constant. Calculate the temperature of platinum thermometer corresponding to 400°C on glass scale. **An**[384.8℃]

1.5: TYPES OF THERMOMETERS

a)-Liquid in glass thermometer;

- Place the bulb in pure melting ice and the length of the mercury column, L_0 is measured and recorded
- Place the bulb in steam from boiling water and the length of the mercury column, L_{100} is measured and recorded
- Place the bulb in contact with the body of an unknown temperature θ and the length of mercury column L_θ is measured and recorded
- Unknown temperature is determined from $\theta = \left(\frac{L_\theta - L_0}{L_{100} - L_0}\right)x100℃$

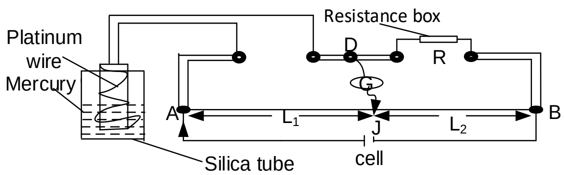
Advantages of a Liquid in Glass Thermometer

- It is easy to use
- It is very cheap
- It is very portable
- It has direct readings

b)-RESISTANCE THERMOMETER [PLATINUM RESISTANCE THERMOMETER]

A resistance thermometer uses resistance(R) of a metal wire as a thermometric property.

MEASUREMENT OF CELCIUS SCALE TEMPERATURE OF A BODY USING PLATINUM RESISTANCE THERMOMETER



- Place the resistance thermometer in a funnel with crushed ice and leave it for some time.
- Obtain the balance point by adjusting the resistance box. The length l_1 and l_2 are read and recorded,
- Determine the resistance of platinum R_0 at 0°C from $R_0 = \frac{l_1}{l_2}R_s$
- Transfer the resistance thermometer to a beaker containing water which is gradually heated to boiling point obtain the new balance point and determine R_{100}
- Immerse the coil in water at room temperature θ and resistances R_θ obtained
- Unknown temperature, $\theta = (\frac{R_\theta - R_0}{R_{100} - R_0})$ x100°C

DISADVANTAGES OF PLATINUM RESISTANCE THERMOMETERS

- It cannot measure very rapidly changing temperature. This is because it has low -thermal conductivity and high heat capacity.
- It cannot measure temperature at a point due to size of silica tube.
- Its heavy and not portal

C) -THERMO COUPLE THERMOMETER

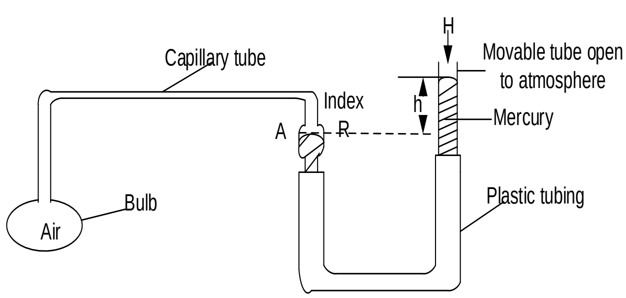
ADVANTAGES OF THERMO COUPLE

- It measures temperature at a point e.g. temperature of crystal since the wires can be made thin.
- It is used to measure rapidly changing temperatures. This is because of its small heat capacity and high thermal conductivity.
- It is portable
- It has a wide range of temperature between -250°C to 1600°C and this can be achieved by using different metals.
- It can be used to determine direct readings if connected to galvanometer which has been calibrated to read temperatures directly.

DISADVANTAGES OF THERMO COUPLE

- It cannot measure slowly changing temperatures.
- It is inaccurate because $E.m.f$ doesn't vary linearly with temperature.

d)-CONSTANT VOLUME GAS THERMOMETER



Corrections in a constant volume gas thermometer include;

- The temperature of the gas in the dead space because its temperature lies between that of the bulb and the room temperature.
- Thermal expansion of the bulb
- The capillary effect at the mercury surface.

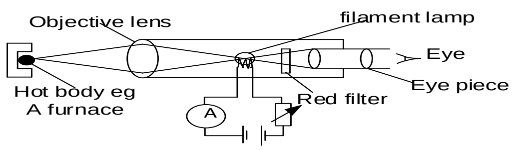
e)-PYROMETERS

They are used to measure very high temperatures e.g. temperature of furnace

They are divided into two;

- Total radiation pyrometer which responds to total radiation i.e. heat and light produced.
- Optical radiation pyrometer which responds to only light produced.

OPTICAL RADIATION PYROMETER



- The filament is focused by the eye piece and the hot body is focused by objective lens so that the image of the object lies in the <u>same plane</u> as the filament.
- The light from both the filament and the body is passed through red filter and viewed by the eye piece.
- Using the rheostat R, the current through filament is adjusted until the filament and object are equally bright. The temperature of the hot body is then read from the ammeter calibrated in °C.

UNEB 2020 Qn5

(a) Define the following

(i) triple point of water (01mark)

(ii) absolute zero temperature (01mark)

(b) Explain why triple point of water is taken as a standard in modern thermometry instead of ice and steam points (04marks)

(c) **(i)** What is a **thermometric property?.** (1mark)

(ii) State **three** qualities of a good thermometric property (03marks)

(d) **(i)** A constant volume thermometer was used to measure temperature when the atmospheric pressure was 760mmHg. The following values were obtained

	Length of mercury in closed limb (mmHg)	Length of mercury in open limb (mmHg)
Bulb in ice	140	130
Bulb in steam	140	330
Bulb at room temperature	140	170

Calculate the room temperature **An**(20°C) (05marks)

(ii) List **three** advantages of the constant volume gas thermometer over the mercury in glass thermometer (02marks)

(e) Explain what happens when the temperature of a fixed mass of ice is raised from 0°C to 10°C (03marks)

UNEB 2020 Qn7

(a) **(i)** Explain how a thermocouple is used to measure temperature on a celcius scale. (5marks)

(ii) Statet **two** advantages of a thermocouple (01mark)

UNEB 2019 Qn7

(d) **(i)** Define a thermometric property. (1mark)

(ii) Describe how a liquid-in-glass thermometer can be used to measure temperature in degrees celcius (04marks)

(i) A thermometer is constructed with a liquid which expands according to relation

$$V_t = V_0(1 + \alpha t + \beta t^2)$$

Where V_t is the volume at t°C and V_0 is the volume at 0°C on the scale of the gas thermometer and α and β are constants. Given that $\alpha = 1000\beta$, what will the liquid thermometer read when the gas thermometer reads 50°C **An(47.73°C)** (04marks)

UNEB 2018 Q5

(a) Define

(i) Thermometric property (01mark)

(ii) Specific heat capacity (01mark)

(b) (i) State **two** examples of commonly used thermometric properties. (01mark)

(ii) Describe briefly how to determine the lower and upper fixed points for an un-calibrated liquid-in-glass thermometers. (04marks)

UNEB 2017 Qn5

(a) (i) State the thermometric property used in the constant-volume gas thermometer (1marks)

(ii) Give **two** characteristics of a good thermometric property (02marks)

(b) (i) Describe the steps taken to set up a Celcius scale of temperature for a mercury-in-glass thermometer (04marks)

(ii) State four disadvantages of mercury-in-glass thermometer. (02marks)

(c) Describe with the aid of a labelled diagram the operation of an optical pyrometer. (06marks)

(d) When oxygen is withdrawn from a tank of volume $50l$, the reading of a pressure gauge attached to the tank drops from $21.4x10^5 Pa$ to $7.8x10^5 Pa$. If the temperature of gas remaining in the tank falls from $30°C$ to $10°C$, calculate the mass of oxygen withdrawn. **An(828.8g)** (05marks)

UNEB 2015 Qn5

(e) (i) State four desirable properties a material; must have to be used as a thermometric substance

(ii) State why scales of temperature based on different thermometric property may not agree

UNEB 2011 Qn 5

(b) (i)Define the term thermometric property and give four examples (02marks)

(ii) State two qualities of a good thermometer property (01marks)

(c) (i)With reference to the a liquid in glass thermometer, describe the steps involved in setting up a Kelvin scale of temperature (03marks)

(ii) State one advantage and disadvantage of the resistance thermometer. (01mk)

(d)A resistance thermometer has a resistance of 21.42Ω at ice point, 29.10Ω at steam point and 28.11Ω at some unknown temperature θ. Calculate θ on the scale of this thermometer. **An[87.11°C]** (03mk)

UNEB 2005 Qn 5

(a) (i)What is meant by the term fixed points in thermometry. Give two examples of such points

(ii) How is temperature on a Celsius scale defined on a platinum resistance thermometer?

(b) Explain the extent to which thermometer based on different properties but calibrate using the same fixed points are likely to agree when used to measure a temperature

(i) Near one of the fixed points (02marks)

(ii) Midway between the two fixed points (02marks)

(d) What are the advantages of a thermocouple over a constant volume gas thermometer in measuring temperature.

Solution

b)i) They may agree, because for points near the fixed points the values of the thermometric properties vary almost in step for points close to the fixed points.

ii) They may not agree for temperature midway between fixed points the different thermometric properties vary differently with temperature.

CHAPTER2: CALORIMETRY

The heat energy of a system is its internal energy and it can be either heat capacity or latent heat.

2.1.0: HEAT CAPACITY AND SPECIFIC HEAT CAPACITY

i) Heat capacity, C

This is the heat required to raise the temperature of a substance by 1°C or 1K.

$$Heat\ capacity = \frac{Quantity\ of\ heat}{Change\ in\ temperature} \qquad C = \frac{Q}{\Delta\theta}$$

The SI unit of heat capacity is Joules per Kelvin [JK^{-1}]

ii) Specific heat capacity, c

This is the heat required to raise the temperature of 1kg mass of a substance by 1°C or 1K.

$$S.H.C = \frac{Quantity\ of\ heat}{Mass\ x\ change\ in\ temperature} \qquad c = \frac{Q}{m\ x\ \Delta\theta}$$

The SI unit of specific heat capacity is Joules per kilogram per Kelvin [$Jkg^{-1}K^{-1}$].

Examples:

1. How much heat is required to raise the temperature of 5kg of iron from 30°C to 40°C if the specific heat capacity of iron is 440 $Jkg^{-1}K^{-1}$?

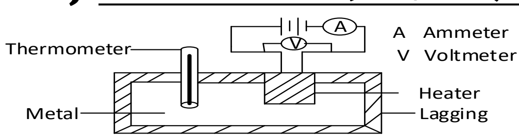
   Solution

   $Heat = mc\Delta\theta$ | $Heat = 5\ x\ 440\ (40 - 30)$ | = 22000J

2. When a block of iron of mass 2kg absorbs 19KJ of heat its temperature rises by 10°C. Find the specific heat capacity of iron

   Solution

a) Determination of S.H.C of a solid by electrical method



- Two holes are drilled into the solid, one for thermometer and other for an electric heater and the holes fikked with mercury for good thermal contact.
- The mass, m of the solid is measured and recorded.
- The apparatus is insulated initial temperature θ_1 recorded.
- A suitable steady current is switched on and at same time stop clock is started. Ammeter and voltage readings I and V are noted.
- When the temperature has risen appreciably, the current is stopped and the time, t of heating is noted and also the final temperature θ_2 is read and recorded.
- Therefore the specific heat capacity, C of the metal is got from

$$C = \frac{Ivt}{m[\theta_2 - \theta_1]}$$

Examples

1. A steady current of $12\ A$ and $p.d$ of $240\ V$ is passed through a block of mass $1500g$ for $1\frac{1}{2}\ minutes$. If the temperature of the block rises from $25°C$ to $80°C$. Calculate;
   (i) S.H.C of the block | (ii) The heat capacity of 4 kg mass of the block

   Solution

(i) Using continuous flow method

- A steady flow of the liquid is set and system left to run until thermometers indicate steady temperatures.
- The inflow temperature θ_1 and out flow temperature θ_2 are read and recorded
- The Ammeter reading I_1 and Voltmeter reading V_1 are read and recorded
- The mass m_1 which flows per second is measured and recorded

- At steady state $I_1V_1 = m_1c(\theta_2 - \theta_1) + h$-- [1] where h is rate of heat loss to surrounding.
- The experiment is repeated for different flow rate. The current and voltage are adjusted until the inflow and outflow temperatures are the same as before
- The Ammeter reading I_2 and Voltmeter reading V_2 are read and recorded
- The new mass m_2 which flows per second is measured and recorded
- At steady state $I_2V_2 = m_2c(\theta_2 - \theta_1) + h$ - [2] Therefore specific heat capacity of a liquid, c is got from

$$C = \frac{I_2V_2 - I_1V_1}{(m_2 - m_1)(\theta_2 - \theta_1)}$$

Examples

1) In continuous flow experiment it was found that when applied $p.d$ was 12.0V, current 1.5A, a rate of flow of liquid of 50.0g/minute cause the temperature of inflow liquid to differ by 10°C. When the $p.d$ was increased to 16.0V with current of 1.6A, the rate of flow of 90.0g/minute was required to produce the same temperature difference as before. Find ;
(i) S.H.C of the liquid
(ii) Rate of heat loss to the surrounding

Solution

$$I_1V_1 = m_1c(\theta_2 - \theta_1) + h$$
$$I_2V_2 = m_2c(\theta_2 - \theta_1) + h$$
$$C = \frac{I_2V_2 - I_1V_1}{(m_2 - m_1)(\theta_2 - \theta_1)} = \frac{12x1.5 - 16x1.6}{\left(\frac{50x10^{-3}}{60} - \frac{90x10^{-3}}{60}\right)(10)}$$
$$C = 1.14x10^3 J\,kg^{-1}\,K^{-1}$$

ii) $I_2V_2 = m_2c(\theta_2 - \theta_1) + h$
$$16x1.6 = \frac{90x10^{-3}}{60}x1.14x10^3x10 + h$$
$$h = 8.50watts$$

2) In the flow method to determine the S.H.C of the liquid, the following two sets of results were obtained.

	Experiment 1	Experiment 2
P.d across water (V)	5.0	3.0
Current through heater (A)	0.3	0.2
Temperature of liquid at inlet (°C)	25	25
Temperature of liquid at outlet (°C)	41	41
Mass of liquid (kg)	0.15	0.07
Time taken (s)	200	120

a) Calculate the S.H.C of the liquid
b) Heat lost per second

Solution

a) $I_1V_1 = m_1c(\theta_2 - \theta_1) + h$
$I_2V_2 = m_2c(\theta_2 - \theta_1) + h$

$$C = 3.3x10^2 J\,kg^{-1}\,K^{-1}$$

b)
$$h = -2.55J$$

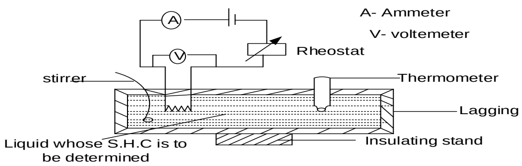
EXERCISE: 39

1) A 15W heating coil is immersed in 0.2kg of water and switched on for 560 seconds during which time the temperature rises by 10°C. Calculate the S.H.C of the liquid. **An [3100 Jkg^{-1}K^{-1}]**
2) With a certain liquid, the inflow and outflow temperatures were maintained at 25.20°C and 26.51°C respectively for a p.d of 12.0V and current 1.50A, the rate of flow was 90g per minute, with 16.0V and 2.00A, the rate of flow was 310g per minute. Find the S.H.C. of the liquid and also the power lost to the surrounding. **An [2910 Jkg^{-1}K^{-1}, 12.3W]**
3) In a continuous flow method, the inflow and outflow temperatures were maintained at 17.0°C and 22.0°C respectively for a p.d of 6.0V and current 2.1A, the rate of flow was 35g per minute, with 4.0V and 1.4A, the rate of flow was 15g per minute. Find the S.H.C. of the liquid and also the rate of loss of heat to the surrounding. **An [4.2Jkg^{-1}K^{-1}, 0.35W]**
4) A student uses continuous flow experiment to determine the specific heat capacity of water. The first experiment was done with a flow rate of 40g per minute and a power input of 30W. The steady state readings on the two thermometers were 18.5°C for the inlet water temperature and 26.5°C for the outlet water temperature. When the flow rate was adjusted to 20 g per minute and power input of 18.25W was found to give the same temperature difference as before.
An [4406 Jkg^{-1}K^{-1}]
5) In determination of the specific heat capacity of water using the continuous flow method, the following results were taken;

	Experiment 1	Experiment 2
P.d across water (V)	3.05	3.15
Current through heater (A)	6.55	7.54
Temperature of liquid at inlet (^oC)	30	30
Temperature of liquid at outlet (^oC)	41.5	41.5
Mass of liquid (kg)	0.431	0.524
Time taken (minutes)	20	20

Calculate the S.h.c of water. **An [4234 Jkg^{-1}K^{-1}]**

(ii) **S.H.C of a liquid using Electrical method**



- A liquid of mass, m is poured in a copper calorimeter of mass, m_c and specific heat capacity, c_c
- The temperature, θ_1 of the liquid is then recorded from the thermometer immersed in the liquid
- A suitable steady current is switched on and stop clock is started simultaneously. Ammeter and voltage readings I and V are noted.
- When the temperature has risen appreciably, the current is stopped and the time, t of heating is noted and also the final temperature θ_2 is read and recorded.

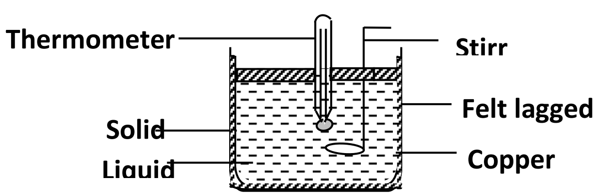
$$IVt = mc(\theta_2 - \theta_1) + m_c c_c(\theta_2 - \theta_1)$$

$$c = \frac{IVt - m_c c_c(\theta_2 - \theta_1)}{m(\theta_2 - \theta_1)}$$

- Hence specific heat capacity, c of the solid can be calculated

METHOD OF MIXTURES

a) For a solid



- A solid of mass, m_s whose specific heat capacity, c_s is required is heated to a temperature, θ_3
- A solid is then transferred quickly to a copper calorimeter of mass, m_c and specific heat capacity c_c containing water of mass, m_w at a temperature, θ_1.
- The mixture is well stirred until a maximum temperature, θ_2 is reached.
- Hence specific heat capacity, c_s of a solid can be calculated

$$c_s = \frac{m_w c_w(\theta_2 - \theta_1) + m_c c_c(\theta_2 - \theta_1)}{m_s(\theta_3 - \theta_2)}$$

Examples

1. The temperature of 500g of a certain metal is raised to 100℃ and it is then placed in 200g of water at 15℃. If the final steady temperature rises to 21℃, calculate the S.H.C of the metal.

Solution

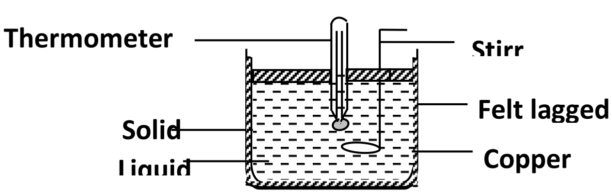
Heat lost by metal = heat gained by water

$$M_m C_m(100 - 21) = M_w C_w(21 - 15)$$

$$0.5x\ C_m x(100 - 21) = 0.2x4200x(21 - 15)$$

$$C_m = 128\ J\ kg^{-1}\ K^{-1}$$

b) For a liquid



- A solid of mass, m_s and specific heat capacity, c_s is heated to a temperature, θ_3
- A solid is then transferred quickly to a copper calorimeter of mass, m_c and specific heat capacity c_c containing a liquid of mass, m_l at whose specific capacity, c_l at temperature, θ_1 is required.
- The mixture is well stirred until a maximum temperature, θ_2 is reached.
- Assuming there are no heat loses during the experiment

$$c_l = \frac{m_s c_s(\theta_3 - \theta_2) - m_c c_c(\theta_2 - \theta_1)}{m_s(\theta_2 - \theta_1)}$$

- Hence specific heat capacity, c_l of the liquid can be calculated

Examples

1. What is the final temperature of the mixture if 100g of water at $70°C$ is added to 200g of cold water at $10°C$. And well stirred (Neglect the heat absorbed by the container and S.H.C of water is 42000$J\ kg^{-1}\ K^{-1}$) .

Solution

Heat lost by hot water = heat gained by cold water

$$M_H C_H(\theta_1 - \theta_3) = M_C C_C(\theta_3 - \theta_2)$$

$$100x10^{-3}(70 - \theta) = 200x10^{-3}(\theta - 10)$$

$$\theta = 30°C$$

2.1.3: COOLING CORRECTION

Is the number of degree Celsius that should be added to the observed maximum temperature to cater for heat losses during rise or fall.

OR

Is the extra temperature that is added to the observed maximum temperature to compensate for the heat lose to the surrounding.

2.1.4: DETERMINATION OF COOLING CORRECTION OF A POOR CONDUCTOR E.G. RUBBER

Question: Explain why a small body cools faster than larger bodies of the same material.

Rate of heat loss $\propto \frac{surface\ area}{volume}$. This implies that heat loss$\propto \frac{1}{length}$. Since $\frac{d\theta}{dt} = -{}^{1}/_{mc}\frac{dQ}{dt}$ and mass$\propto$ $volume$, a small body cools faster than a large body

2.1.5: NEWTON'S LAW OF COOLING

It states that under conditions of forced convection, the rate of heat loss is directly proportional to excess temperature over the surrounding

$$\frac{dQ}{dt} \propto (\theta - \theta_R),$$

$$\frac{dQ}{dt} = -k(\theta - \theta_R),$$

But $\frac{dQ}{dt} = mc\frac{d\theta}{dt}$

$$\therefore \frac{d\theta}{dt} = -k(\theta - \theta_R)$$

EXPERIMENT TO VERIFY NEWTON'S LAW OF COOLING

- Hot water in a calorimeter is placed near an open window.
- Temperature θ of the water is recorded at suitable time intervals
- A graph of temperature θ against time t is plotted.
- Different slopes at different temperatures θ_1,θ_2, θ_3..........θ_A are determined.
- For each temperature the excess temperature, $(\theta - \theta_R)$is calculated, where θ_R is room temperature
- A graph of slope against excess temperature is plotted
- A straight line graph through the origin verifies Newton's law of cooling.

Why temperature remains constant during change of state (phase)

- During melting (change of state from solid to liquid), the heat energy supplied is used to weaken the intermolecular forces and increase separation between molecules. This increases the potential energy of the molecules but the mean kinetic energy of the molecules remain constant . Further increase in separation between molecules causes the regular patterns to collapse as the solid changes to a liquid, until the process is complete the temperature remains constant.

- During boiling (change from liquid to vapour state) the heat supplied is used to break the intermolecular forces and increases separation between molecules. This increases the potential energy of the molecules but the mean kinetic energy of the molecules remain constant . Also some of the energy is used in doing work during expansion against atmospheric pressure, hence no temperature change occurs.

Significance of latent heat on regulation of body temperature

SPECIFIC LATENT HEAT OF FUSION

Is the quantity of heat required to change **1kg** mass of a solid to a liquid at **constant temperature**.

It's unit is Jkg^{-1}

LATENT HEAT OF VAPOURIZATION

Is the quantity of heat required to change any mass of substance from liquid to gas at a constant temperature.

2.2.1: WHY LATENT HEAT OF VAPOURIZATION IS HIGHER THAN LATENT HEAT OF FUSION

Examples

1. A calorimeter with heat capacity of $80J°C^{-1}$contains 50g of water at 40°C what mass of ice at 0°C needs to be added in order to reduce the temperature to 10°C. Assume no heat is lost to the surround (S.H.C of water = 4200J$kg^{-1}°C^{-1}$, S.L.H of the of ice = 3.4x10^{5}Jkg^{-1}).

Solution

$$80\ x\ (40-10)\ +0.05\ x4200x(40-10)\ = m_I\ (3.4x10^5+4200x10)$$
$$m_{\ I} =\ 0.023kg$$

2.2.2: DETERMINATION OF THE S.L.H OF VAPOURIZATION (L_v) OF LIQUID:
a)ELECTRIC METHOD [DEWAR FLASK METHOD]

Examples

1) When electrical energy is supplied at a rate of 12W to a boiling liquid 5.0x10^{-3} Kg of the liquid evaporates in 30 minutes .On reducing the electrical power to 7W, 1.0x10^{-3} Kg of the liquid evaporates in the same time. Calculate;
   a) S.L.H of vapourisation
   b) Power loss to the surrounding

Solution

$I_1\ V_1\ t\ = m_1 \text{x}\ l_V + h,\quad I_2\ V_2\ t\ = m_2 \text{x}\ l_V + h$

$$L_V = \frac{(I_2V_{2-}\ I_1V_1)t}{(M_{2-}\ M_1)} = \frac{(7-12)x30x60}{(1x10^{-3}-5x10^{-3})}$$

$Lv = 2.25x10^6Jkg^{-1}$

b) $I_1\ V_1\ = \frac{m_1}{t} \text{x}\ l_V + h$

$$12\ = \frac{5x10^{-3}}{30x60}x\ 2.25x10^6 + h$$

h= $5.75W$

2) In an experiment to determine S.L.H.V of a liquid using Dewar flask in the following results were obtained.

Voltage V(V)	Current I(A)	Mass collected in 300s/g
7.4	2.6	5.8
10.0	3.6	11.3

Calculate the;

a) S.L.H of vapourization of alcohol

b) Average rate of heat loss to the surrounding

Solution

$I_1 V_1 t = m_1 \text{x} l_V + h \ldots\ldots\ldots\ldots\ldots (i)$,

$I_2 V_2 t = m_2 \text{x} l_V + h \ldots\ldots\ldots\ldots.. (ii)$

$Lv = \frac{I_2V_2 - I_1V_1}{M_2 - M_1} = \frac{10x3.6 - 7.4x2.6}{(11.3-5.8)x\frac{1}{300}x10^{-3}}$

$Lv = 9.14x\,10^5\,Jkg^{-1}$

Put into equation (2): $I_2 V_2 t = m_2 \text{x} l_V + h$

$10x\,3.6 = \frac{11.3}{300}x10^{-3}x9.14x10^5 + h$

$h = 1.57W$

Excersise:41

1) A student performs two experiments to measures the specific latent heat of ethanol using an electrical method.

Experiment 1	Experiment 2
V_1=8.90V	V_2=7.30V
I_1=2.10A	I_2=1.74A
m_1=174g	m_2=111g
t_1=15 minutes	t_2=15 minutes

Calculate the specific latent heat of ethanol. **An $8.55x10^4\,Jkg^{-1}$**

2) In an experiment to determine S.L.H.V of a liquid using Dewar flask, the following results were obtained.

Voltage V(V)	Current I(A)	Mass collected in 400s/g
10.0	2.00	14.6
11.2	2.50	30.6

Calculate the heat lost to surrounding 400s. **An(5080J)**

3) In an experiment to determine S.L.H.V of a liquid at its boiling point. The following results were obtained.

Voltage V(V)	Current I(A)	Mass (g) evaporated in 400s
10.0	2.00	14.6
15.0	2.50	30.6

Calculate the;

a) S.L.H of vapourization of liquid
b) Energy loss to the surrounding in 400s
c) Rate of evaporation of the liquid when a 30.0W rate of heating is used. **An $4.38x10^5\,Jkg^{-1}$, 1.61kJ, $0.0594gs^{-1}$**

b)DETERMINATION OF S.L.H.V BY METHOD OF MIXTURE

- The mass m_1 of water and the calorimeter is measured and noted
- The initial temperature, θ_1 of water in the calorimeter is noted
- Steam from boiling water is then passed into the water in the calorimeter through a steam trap.
- After a measurable temperature rise, the final temperature, θ_2 of the water in calorimeter is measured and noted.
- The new mass, m_2 of the water and the calorimeter is again measured and the mass, m_s of condensed steam is calculated from $m_s = m_2 - m_1$
- Temperature θ_3 of steam is measured by thermometer T and recorded
- The mass m_c of the empty calorimeter is obtained by weighing

$$\begin{pmatrix}\text{Heat given by}\\ \text{steam condensing}\end{pmatrix} + \begin{pmatrix}\text{heat given by}\\ \text{condensed water}\\ \text{from } \theta_3 \text{ to } \theta°C\end{pmatrix} = \begin{pmatrix}\text{heat taken}\\ \text{by calorimeter}\end{pmatrix} + \begin{pmatrix}heat\ taken\\ by\ water\end{pmatrix}$$

$m_s\,l_v + m_s\,C_w\,(\theta_3 - \theta_2) = (m_cC_c + m_wC_w)(\theta_3 - \theta_2)$

C_w = S.H.C of water

m_w = mass of water $where\ m_w = m_1 - m_c$

C_c = S.H.C of calorimeter

- l_v can be obtained

Examples

1) An electric kettle with a $2.0kW$ heating element has a heat capacity of 400JK. 1.0kg of water at 20°C is placed in the kettle. The kettle is switched on and it is found that 13 minutes later the mass of water in it is 0.5kg. Ignoring heat losses calculate a value for the specific latent heat of vaporization of water. (specific heat capacity of water is 4200 $Jkg^{-1}K$)

Solution

$$Pt = m_f\,C_f\,(\theta_2 - \theta_1) + m_w\,C_w\,(\theta_2 - \theta_1) + m_s l_v$$

$$2x1000x13x60 = 400\,(100 - 20) + 1x4200[100 - 20] + (1 - 0.5)l_v$$

$$l_v = 2.38x10^6 Jkg^{-1}$$

Explain why specific latent heat of vaporization of water is higher at 20°C than at 100°C

- At 20°C the molecules of the liquid are closer together than at 100°C. The intermolecular forces of attraction are stronger at 20°C than at 100°C.
- More energy is required to break the bonds at 20°C than the heat needed at 100°C

UNEB 2020 Q6

(a) Define **specific heat capacity** (01mark)

(b) Describe, stating the assumptions made, an electrical method for the determination of the specific heat capacity of a metal. (08marks)

(c) In an experiment to determine specific heat capacity of a liquid using the continuous flow calorimeter;

(i) The readings are taken when the apparatus has attained a steady state. Explain the meaning of a steady state (02marks)

(ii) Explain why two sets of reading are taken (01mark)

(d) When water is passed through a continuous flow calorimeter at the rate of 100gmin^{-1}, the temperatures rises from 16°C to 20°C, when the p.d across the heater is 20V and the current is 1.5A. When another liquid at 16°C is passed through the calorimeter at the rate of 120gmin^{-1}, the same temperature change is obtained at a p.d of 13V and current 1.2A. Calculate the S.H.C of the liquid. (4marks) **An[1700** Jkg^{-1}K^{-1}**]**

(e) **(i)** Define **latent heat** (01mark)

(ii) Explain why latent heat if vaporization is always greater than that of fusion (02marks)

UNEB 2019 Q5

(c) **(i)** Describe an electrical method of determining the specific heat capacity of a good conducitng solid (06marks)

(ii) Give any two reasons why the value obtained using the method in (c)(i) may not be accurate (02marks)

UNEB 2018 Q5

(c) (i) Describe with the aid of a diagram, an experiment to determine the specific heat capacity of a liquid using the continuous flow method. (07marks)

(ii) State **two** advantages of the continuous flow method over the method of mixture. (01mark)

(ii) State **two** disadvantages of the method in (c) (i). (01mark)

(b)The brake linings of the wheels of a car of mass 800kg have a total mass of $4.8kg$ and are made of a material of specific heat capacity $1200\ J\ kg^{-1}\ K^{-1}$. If the car is moving at 15ms^{-1} and is brought to rest by applying the brakes, calculate the maximum possible temperature rise of the brake linings. (04marks)

$^1/_2 Mv^2 = mc\Delta\theta$ | $^1/_2\, x\ 800x15^2 = 4.8x1200\Delta\theta$ | $\Delta\theta = 15.6$°C

UNEB 2016 Q5

(a) (i) Define **specific latent heat of fusion** (01mark)

(ii) State the effect of impurities on melting point. (01mark)

(b) Explain why there is no change in temperature when a substance is melting (04marks)

(c) With the aid of a diagram, describe the continuous flow method of measuring the specific heat capacity of a liquid (06marks)

(d) In an experiment to determine the specific heat of fusion of ice, a heating coil is placed in a filter funnel and surrounded by lumps of ice. The following two sets of readings were obtained.

V(V)	4.0	6.0
I(A)	2.0	3.0
Mass of water m(g) collected in 500 s	14.9	29.8

Calculate the;

(i) Specific latent heat of fusion of ice. **An [$3.36x10^5 Jkg^{-1}$]** (04marks)

(ii) Energy gained in the course of obtaining the first set of readings **An [500J]** (03marks)

(e) Why are two sets of readings necessary in (d) above. (01mark)

UNEB 2015 Q5

(c) Describe with the aid a diagram an experiment to determine specific latent heat of vaporization of steam using the method of mixtures (07marks)

(d) A 600W electric heater is used to raise the temperature of a certain mass of water in a thermos flask from room temperature to 80°C. The same temperature rise is obtained when steams from a boiler is passed into an equal mass of water at room temperature in the same time. If 16g of water were being evaporated every minute in the boiler, find the specific latent heat of vaporisation of steam, assumption no heat loses. **An($2.26x10^6$ Jkg^{-1})** (04marks)

UNEB 2014 Q7

(a) Define specific latent heat of vaporisation (01mark)

(b) With the aid of a labelled diagram, describe an experiment to measure the specific latent of vaporisation of a liquid using an electrical method (07mark)

(c) Explain the effect of pressure on boiling point of a liquid (02mark)

(d) A liquid of specific heat capacity $2.8x10^3 Jkg^{-1}K^{-1}$ and specific latent hate of vaporisation $9.00x10^5 Jkg^{-1}$ is contained in a flask of heat capacity 800JK^{-1} at a temperature of 32°C. An electric heater rated 1 kW is immersed in 2.5kg of the liquid and switched on for 12 minutes, calculate the amount of liquid that boils off, given that boiling point of the liquid is 80°C

$\boldsymbol{An(3.84x10^{-1}kg)}$ (06mark)

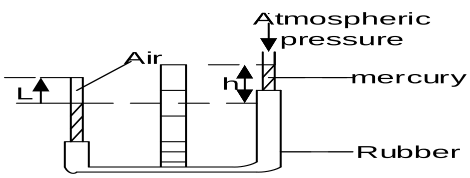
CHAPTER3: GAS LAWS AND GAS PROCESSES

3.1: Boyle's law:

it states that the pressure of fixed mass of a gas is inversely proportional to its volume at constant temperature i.e.

P $\propto \frac{1}{v}$ | PV = constant | $\boxed{P_1V_1 = P_2V_2}$

Experiment to verify Boyles law



- A fixed mass of the gas is trapped inside J tube of uniform cross section using mercury.
- Measure and record the atmospheric pressure H using a barometer
- Adjust the flexible tube by lowering or raising the open end.
- Measure and record the difference in mercury levels h
- Record the length l of the air column trapped in the closed tube
- Obtain the air pressure, $P = H \pm h$.
- Repeat the procedure and obtain a series of values P and l , $l \propto volume$
- Plot a graph of P against $\frac{1}{l}$ and a straight line graph passing through origin verifies Boyles law

Absolute zero temperature (0K)

is the temperature attained when molecules slow down and acquire their minimum possible energy.

Molecular explanation for existence of absolute zero temperature

When a gas is cooled, its molecules loose kinetic energy continuously. As molecules loose kinetic energy they move closer into close proximity until when they cease to have kinetic energy. At this point the gas is said to occupy a negligible volume and its temperature at this point is called the absolute zero temperature and the pressure the gas exerts on the walls of the container occupied is negligible.

Examples

1) A gas is confined in the container of volume 0.1m^3 at pressure of $1.0x10^5Nm^{-2}$ and temperature of 300K. If the gas is found to be ideal gas, calculate the density of the gas [$Rmm = 32$]

Solution

$PV = nRT$

$\therefore n = \frac{pv}{RT} = \frac{1.0x10^5x0.1}{8.31x300}$ = 4.01moles

But $m = nM$

$m = 0.032x4.01$

Mass = $0.128kg$

But $\rho = \frac{M}{V} = \frac{0.128}{0.1} = 1.28kgm^{-3}$

Calculation involving loss of gas

1) Oxygen gas is contained in cylinder of volume $1.0x10^{-2}m^3$ at temperature of 300K and pressure $2.5x10^5Nm^{-2}$. After some oxygen is used at constant temperature, pressure falls to $1.3x10^5Nm^{-2}$ Calculate the mass of oxygen used.

Solution

$V_1 = 1.0x10^{-2}m^3, T_1 = 300K$,

$P_1 = 2.5x10^5Nm^{-2}, M = 32g$ (R.M.M of oxygen)

$PV = \frac{m}{M}R\ T$

$\therefore m_1 = \frac{P_1V_1\ M}{RT_1} = \frac{2.5x10^5x1x10^{-2}x32x10^{-3}}{8.31x300} = 0.032kg$

$V_2 = 1.0x10^{-2}m^3, T_2 = 300K$,

$P_2 = 1.3x10^5Nm^{-2}$

$m_2 = \frac{1.3x10^5x1x10^{-2}x32x10^{-3}}{8.31x300} = 0.0166kg$

Therefore mass of oxygen = [m_1 - m_2] kg

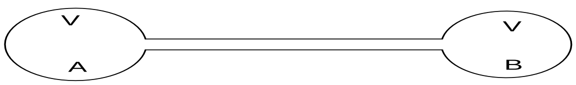
$= [0.032 - 0.0166]\,kg = 0.0154\,kg$

Connected containers

In closed containers the total number of molecules remains constant

1) Two glass bulbs of equal volume are joined by another tube and are filled with a gas at $s.t\,p$. When one of the bulbs is kept in melting ice and another place in as hot bath the new pressure is 877.6mmHg. Calculate the temperature of bath

Solution



P_A = 760mmHg P_B = 760mmHg

T_A = 273k T_B = 273K

Since cylinders are enclosed, the number of moles is both cylinders before cooling will be the same after cooling (heating).

$n_A + n_B = n_A^1 + n_B^1$

$$\frac{P_AV_A}{RT_A} + \frac{P_BV_B}{RT_B} = \frac{P'_AV'_A}{RT'_A} + \frac{P'_BV'_B}{RT'_B}$$

$P'_A = P'_B = 877.6mmHg$

$T'_A = (0 + 273) = 273K$ T'_B =?

$$\frac{760xV}{8.31x273} + \frac{760xV}{8.31x273} = \frac{877.6xV}{8.31x273} + \frac{877.6xV}{8.31xT'_B}$$

$$\frac{642.4}{2268.63} = \frac{877.6}{8.31T'_B}$$

$T'_B = 372.95K$

3.5: Daltons law of partial pressure

It states that the total pressure of a mixture of gases that do not react chemically is the sum of partial pressure of the constituents

Definition. Partial pressure of gas is the pressure the gas would exert if it was to occupy the whole container alone.

Examples

1) Two containers A and B of volume 3x10^{3}cm^3 and 6x10^{3}cm^3 respectively contain helium gas at pressure 1x10^{3}Pa and temperature 300K. Container A is heated to 373Kwhile container B is cooled to 273K. Find the final pressure of the helium gas.

Solution

2) Two cylinder A and B of volume V and 3V respectively are separately filled with gas. The cylinders are connected with tap closed with pressure of gas A and B being P and 4P respectively. When tap is open, the common pressure becomes 60Pa. Find P

Solution

$P = \frac{P_A \cdot V_A}{V_A + V_B} + \frac{P_BV_B}{V_A + V_B}$

$60 = \frac{PxV}{V+3V} + \frac{4Px3V}{V+3V}$

$P = 18.46Pa$

3.8: THE 1ST LAW OF THERMO DYNAMICS

The **1st law states** that the total energy in a closed system is conserved.

When we warm gas so that it expands, the heat (ΔQ) appears partly as an increase in internal energy (Δu) and partly as external work done (Δw).

$$\boxed{\Delta Q = \Delta u + \Delta w}$$

But $\Delta w = P\Delta V$

$\Delta Q = \Delta u + P\Delta V$

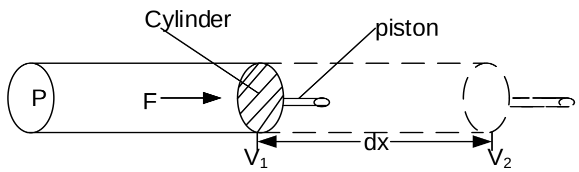
ΔQ = heat supplied

Δu = increase in internal energy

Δw = work done

3.9: Work done by a gas in expansion at constant pressure

For an ideal gas enclosed in a cylinder by a frictionless piston of area of cross-section A. gas expands by pushing piston by dx



Force on piston, F=PA

Work done during expansion gas $dw = Fdx$

$$dw = PAdx$$

$\therefore dw = Pdv$ since $dv = Adx$

$\int_0^W dw = \int_{v_1}^{v_2} Pdv$

W = $\int_{v_1}^{v_2} Pdv$ .... (A)

W = $\int_{v_1}^{v_2} Pdv = \mathrm{P}[v]_{v_1}^{v_2} = \mathrm{P}\,[V_2 - V_1]$ ....(B)

Generally :The external work done in expanding gas at constant pressure $\boxed{W = P\Delta V}$

a) MOLAR HEAT CAPACITY AT CONSTANT VOLUME

Is the amount of heat required to change the temperature of 1mole of gas by 1 Kelvin at constant volume

It is denoted by C_V (C-capital). It is measured in $Jmol^{-1}K^{-1}$

$C_V = c_v M$ Where M = molar mass

b) MOLAR HEAT CAPACITY AT CONSTANT PRESSURE

Is the amount of heat required to change the temperature of 1 mole of gas by 1 Kelvin at constant pressure

It is denoted by C_P (C-capital) and it is measured $Jmol^{-1}K^{-1}$.

$$C_P = c_p M$$

3.11: DIFFERENCES BETWEEN MOLAR HEAT CAPACITIES [$C_P - C_V = R$]

From 1st law of thermodynamics: $\Delta Q = \Delta u + \Delta w$

At constant pressure: nC_P ΔT = Δu + P ΔV .... (1)

For an ideal gas equation P$\Delta V = n$RΔT

nC_P ΔT $= \Delta u + nR\Delta T$

At constant volume $nC_V\Delta T = \Delta u + 0$ since P$\Delta V = 0$

nC_P ΔT $= nC_V\Delta T + nR\Delta T$

$C_P = C_V + R$

$\boxed{C_P - C_V = R}$

Where $\frac{C_P}{C_V} = \gamma$

Examples

1) The density of a gas with S.H.C of a gas at constant pressure of $890Jkg^{-1}K^{-1}$ at a temperature of 20°C and pressure of $1.01x10^5Pa$ is $1.54kgm^{-3}$. Calculate the S.H.C of oxygen at constant volume

Solution

$PV = mrT$ But $m = v\rho$

$$r = \frac{P}{\rho T} = \frac{1.01x10^5}{1.54x293} = 223.84\,Jkg^{-1}K^{-1}$$

$$c_P - c_V = r$$

$$c_v = 890 - 223.84 = 666.16Jkg^{-1}K^{-1}$$

Alternatively

$$PV = \frac{m}{M}RT \text{ But } m = v\rho$$

$$M = \frac{\rho RT}{P} = \frac{1.54x8.31x293}{1.01x10^5} = 0.0371kg$$

But $C_P - C_V = R$

where C_P and C_V are molar heat capacities

$c_pM - c_vM = R$ where c_p and c_v are S.H.C are constant pressure and volume respectively

$$c_v = \frac{890x0.0371 - 8.31}{0.0371} = 666.01\,Jkg^{-1}K^{-1}$$

2) The S.H.C of oxygen at constant volume is $719Jkg^{-1}K^{-1}$ If the density of oxygen at S.T.P is $1.429kgm^{-3}$. Calculate the S.H.C of oxygen at constant pressure

Solution

$PV = mrT$ But $m = v\rho$

$$r = \frac{P}{\rho T} = \frac{1.01x10^5}{1.429x273} = 258.9\,Jkg^{-1}K^{-1}$$

$$c_P - c_V = r$$

$$c_p = 719 + 258.9 = 977.9Jkg^{-1}K^{-1}$$

Alternatively

$$PV = \frac{m}{M}RT \text{ But } m = V\rho$$

$$M = \frac{\rho RT}{P} = \frac{1.429x8.31x273}{1.01x10^5} = 0.0324kg$$

But $\boldsymbol{C_P - C_V = R}$

where $\boldsymbol{C_P}$ and $\boldsymbol{C_V}$ are molar heat capacities

$c_pM - c_vM = R$ where c_p and c_v are S.H.C are constant pressure and volume respectively

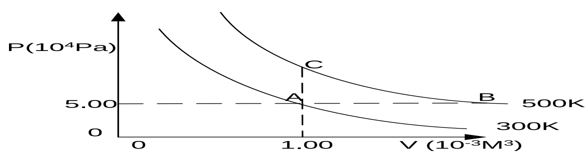
$c_p = \frac{8.31+0.0324x719}{0.0324} = 977.9\,Jkg^{-1}K^{-1}$

EXERCISE:44

1) Nitrogen gas is trapped in the container by movable piston. If temperature of gas is raised from 0°C to 50°C at constant pressure of 4.0x10^5Pa and total heat added is 3.0x10^4 J. Calculate the work done by the gas [Cp=29.1Jmol^{-1}K^{-1}, $\frac{C_P}{C_V}$= 1.4] **(Ans 8.57x10^{3}J).**

2) An ideal gas with volume of 0.1m^3 expands at a constant pressure of $1.5x10^5Pa$ to treble its volume. Calculate the work done by the gas **An(3 $x10^5J$)**

3) The diagram shows curves relating pressure, P and volume V for a fixed mass of an ideal monatomic gas at 300K and 500K. The gas is in a container fitted with a piston which can move with negligible friction.



(a) Give the equation of state for n moles of an ideal gas, defining the symbols used.

(b) Show by calculation that;
   (i) The number of moles of gas in the container is 2.01x10^{-2}
   (ii) The volume of the gas at B on the graph is 1.67x10^{-3}m^3, R= 8.31Jmol^{-1}k^{-1}

4) A steel pressure vessel of volume 2.2x10^{-2}m^3 contains 4.0x10^{-2}kg of a gas at a pressure of 1.0x10^{5}Pa and temperature 300K. An explosion suddenly releases 6.48x10^{4}J of energy, which raises the pressure instantaneously to 1.0x10^{6}Pa. Assuming no loss of heat to the vessel, and ideal gas behavior calculate;
   (a) The maximum temperature attained
   (b) The two principal specific heat capacities of the gas.
   (c) What is the velocity of sound in this gas at a temperature of 300K? **An[3000K, 600Jkg^{-1}K^{-1}, 783 Jkg^{-1}K^{-1}, 268ms^{-1}]**

a)ISOTHERMAL PROCESS

Is the change (expansion or compression) which occurs at constant temperature

For an isothermal change PV = constant. Heat must be supplied at the same rate as the gas is doing its work

$$\Delta Q = \Delta u + \Delta w$$

But $\Delta u = nCv\Delta T$ and $\Delta T = 0 \Rightarrow \Delta u = 0$

$\therefore \Delta Q = \Delta w$ .... (x)

Equation (x) above implies that in an isothermal change all heat supplied to gas must be used to do external work.

REVERSIBLE ISOTHERMAL CHANGE:

It's defined as, a change that occurs at constant temperature and can be made to go in the reverse direction by an infinitesimal change in the conditions causing it to take place

WORK DONE (ΔW) IN AN ISOTHERMAL EXPANSION

Consider an isothermal expansion from $V_1$to V_2

$$\Delta w = P\Delta v$$

$\int_o^w \Delta w = \int_{V1}^{V2} \mathrm{P\Delta v}$

W= $\int_{v1}^{v2} \mathrm{P\Delta v}$

But $P = \frac{\mathrm{nRT}}{V}$

W= $\int_{V1}^{V2} \frac{\mathrm{nRT}}{V}$ dv

W= $nRT \int_{V_1}^{V_2} \frac{1}{V}$ dv

W= $nRT\,[1nV]_{V_1}^{V_2}$

W= nRT (1nV$_2$ -1nV$_1$)

$\boxed{W = n\,R\,T\,In\frac{V_2}{V_1}}$ OR $\boxed{W = P_1\,V_1 In\frac{V_2}{V_1}}$ OR $\boxed{W = P_2\,V_2 In\frac{V_2}{V_1}}$

b)ADIABATIC PROCESS (ΔQ = 0)

An adiabatic process is a change (expansion or compression) in which there is no heat exchange between the gas and the surrounding.

Using the 1st law of thermal dynamics.

$\Delta Q = \Delta u + \Delta w$

But $\Delta Q = 0$

Therefore $\Delta u = -\Delta w$ .... (xx)

- Equation (xx) shows that, in an adiabatic process the external work done in expanding the gas is at expense of internal energy and this result into cooling of the gas.

Question; explain why an adiabatic expansion results into cooling of the gas.

During an adiabatic expansion, no heat is supplied to the gas. Molecules of the gas strike the receding piston and bounce off with reduced velocities hence lower kinetic energies. Since the absolute temperature is proportional to mean kinetic energy of the molecules, the gas cools during expansion

EQUATION FOR ADIABATIC PROCESS

From the 1st law of thermal dynamics

ΔQ = $\Delta u + \Delta w$ .... (1)

But $\Delta u = C_v\Delta T$ for 1mole of gas And $\Delta w = P\Delta V$

Putting these into equation 1

$$\Delta Q = C_v\Delta T + P\Delta V$$

But for an a adiabatic process ΔQ = 0

Therefore $C_v\Delta T + P\Delta V = 0$ ....(2)

$Pv = RT$ for 1mole of an ideal gas

Differentiating it partially, gives

$$P\Delta V + V\Delta P = R\Delta T$$

$P\Delta V = R\Delta T - V\Delta P$ ....(3)

Putting equation (3) into (2), gives

$C_v\Delta T + R\Delta T - V\Delta P = 0$ ....(4)

But $C_P - C_v = R$

$$C_v\Delta T + (C_P - C_v)\Delta T - V\Delta P = 0$$

$$C_P\Delta T - V\Delta P = 0 \ldots\ldots\ldots\ldots.(5)$$

From equation (2) : $\Delta T = \frac{-P\Delta v}{C_v}$

Putting ΔT into (5) : $C_P x(-\frac{P\Delta V}{C_v}) - V\Delta P = 0$

$$\frac{C_P}{C_v} P\Delta V + V\Delta P = 0$$

But $\frac{C_P}{C_v} = \gamma$

$\gamma P\Delta V + V\Delta P = 0$

Driving all through by PV

$\frac{P\gamma\Delta V}{PV} + \frac{V\Delta P}{PV} = 0$

$\frac{\gamma\Delta V}{V} + \frac{\Delta P}{P} = 0$

Integrating : $\gamma\int\frac{\Delta V}{V} + \int\frac{\Delta P}{P} = constant$

$\gamma 1nV + 1nP = constant$

$$1nV^{\gamma} + InP = Inc$$

In$PV^{\gamma} = Inc$

PV^{γ} = Constant

$\boxed{P_1V_1^{\gamma} = P_2V_2^{\gamma}}$

WORK DONE (ΔW) IN AN ADIABATIC EXPANSION

$\Delta Q = \Delta u + \Delta w$ But $\Delta Q = 0$

Therefore $\Delta u = -\Delta w$

$$\Delta u = C_v\Delta T$$

$\Delta w = -nC_v(T_2 - T_1)$.......(1)

But $C_P - C_v = R$

$$\frac{C_p}{C_v} - \frac{C_v}{C_v} = \frac{R}{C_v}$$

$$\gamma - 1 = \frac{R}{C_v}$$

$C_v = \frac{R}{\gamma-1}$..............(2)

From $PV = nRT$

$\Rightarrow \boldsymbol{T_2} = \frac{P_2V_2}{nR}$............(3)

$T_1 = \frac{P_1V_1}{nR} \ldots\ldots\ldots\ldots(4)$

Putting 2, 3, 4 into 1: $\Delta w = -n\frac{R}{\gamma-1}\left(\frac{P_2V_2}{nR} - \frac{P_1V_1}{nR}\right)$

$$\Delta w = -\frac{(P_2V_2 - P_1V_1)}{\gamma - 1}$$

Examples

1) An ideal gas at 18°C is compressed adiabatically until its volume is halved. Calculate the final temperature of gas (assume S.H.C of gas at constant pressure and volume are 2100$Jkg^{-1}K^{-1}$ and 1500 $Jkg^{-1}K^{-1}respectively$)

Solution

$T_1 = (18 + 273) = 291K$

$T_2 = ?,\ V_1 = V,\ V_2 = \frac{V}{2}$

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

But $\gamma = \frac{C_P}{C_v} = \frac{2100}{1500} = 1.4$

$291xV^{1.4-1} = T_2\left(\frac{V}{2}\right)^{1.4-1}$

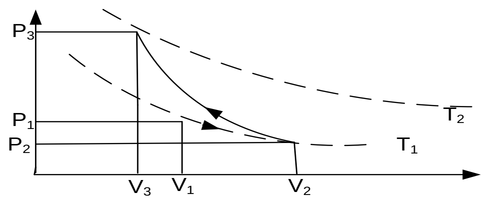
$291xV^{0.4} = T_2\frac{V^{0.4}}{2^{0.4}}$

$T_2 = 383.916K$

2) A gas having a temperature of 27°C volume of 30000cm³ and pressure of 80cmHg expands isothermally to double its volume. The gas is then adiabatically compressed to half its original volume.
   (i) Represent these changes on P-V sketch
   (ii) Calculate final pressure and temperature of gas ($\gamma = 1.4$)

Solution

(i)



(ii) $T_1 = 27 + 237 = 300K$

$V_1 = 3000x10^{-3}$ m³, $V_1 = 3x10^{-3}$m³

$P_1 = 80cmHg$

Isothermally $V_2 = 2V_1 = 6x10^{-3}$

Isothermal $P_1V_1\,T_1 \rightarrow P_2V_2T_1$

$$P_1V_1 = P_2V_2$$

$80x3x10^{-3} = P_2x6x10^{-3}$

$P_2 = 40cmHg$

Adiabatic: $P_2V_2\,T_2 \rightarrow P_3V_3T_2$

But $V_3 = \frac{1}{2}V_2 = 1.5x\ 10^{-3}m^3$

$P_2V_2^{\gamma} = P_3V_3^{\gamma}$

$40(6x10^{-3})^{1.4} = P_3x(1.5x10^{-3})^{1.4}$

$P_3 = 5.092\text{x}10^5Pa$

$P_3 = 278.57cmHg$

$T_1V_2^{\gamma-1} = T_2V_3^{\gamma-1}$

$300x(6x10^{-3})^{0.4} = T_2(1.5x10^{-3})^{0.4}$

$T_2 = 522.3K$

Final pressure = $278.5cmHg$ and final temperature = $522.3K$

UNEB 2011 Q 6

di) Distinguish between isothermal and a adiabatic changes (02marks)

ii)An ideal gas at 18°C is compressed a adiabatically until the volume is halved.

Calculate the final temperature of the gas.

(Assume specific heat capacities of the gas at constant pressure and volume are 2100Jkg⁻¹K⁻¹ and 1500Jkg⁻¹K⁻¹ respectively) **An[383.98k]** (4marks)

UNEB2010 Q.6

a)i)State the difference between isothermal and adiabatic expansion of a gas

ii)Using the same axes and point, sketch the graph of pressure verses volume for a fixed mass of gas undergoing isothermal and a adiabatic change (3marks)

b)Show that the work W done by a gas which expands reversibly from V_0 to V_1 is given by W= $\int_{v0}^{v1} pdv$ (4marks)

c)i)State two differences between real and ideal gases

ii)Draw labeled diagram showing P-V isothermal for a real gas above and below the critical temperature (3mark)

d)Ten moles of a gas initially at 27°C and heated at a constant pressure 1.0x10⁵ Pa and the volume increased from 0.250cm³ to 0.375m³. Calculate the increases in internal energy [assume Cp = 28.5Jmol⁻¹K⁻¹] (6mark) **An [3.012x10⁴J]**

UNEB 2009 Q.6

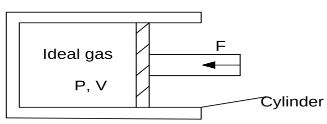
a)i)State Boyle's law (01mark)
ii)Describe an experiment that can be used to verify Boy led law. (06mark)
c)i)What is meant by reversible process
ii)State the conditions necessary for isothermal and adiabatic process to occur
d)A mass of an ideal gas of volume 2000m^3 at 144K expands adiabatically to a temperature of 137K. Calculate the new volume (take γ = 1.40) (3mark) **An [226.47cm^3]**

UNEB 2008 Q.6

a)Describe an experiment to verify Newton's law of cooling
c)ii)Nitrogen gas is trapped in a container by a movable piston. If the temperature of the gas raised from 0°C to 50°C at a constant pressure of $4x10^5$Pa and the total heat added is $3x10^4$J. Calculate the work done by the gas. **An[8.57x10^3]**
(Molar heat capacity of nitrogen at constant pressure is 29.1Jmol^{-1}k^{-1} $\frac{C_P}{C_V} = 1.4$)

UNEB 2007 Q,7

a)



A fixed mass of an ideal gas confined in a cylinder by a friction less piston of cross section area A . The piston is in equilibrium under the action of a force F as shown above. Show that the work done W by the gas when it expands from V_1 to V_2 is given by W= $\int_{v1}^{v2} pdv$
b)State the first law of thermodynamics and use it to distinguish between isothermal and adiabatic changes in a gas.
c)The temperature of one mole of helium gas at a pressure 1.0x10^5Pa increases from 20°C to 100°Cwhen the gas is compressed adiabatically. Find the final pressure of the gas (take $\gamma = 1.67$)
An [1.83x10^{5}Pa]

UNEB 2001 Q.6

a)i)Explain what happens when a quantity of heat is applied to a fixed mass of gas (02marks)
ii)Derive the relation between the principal molar heat capacities Cp and Cv for an ideal gas (05marks)
b)i)What is an adiabatic process (1mark)
ii)A bicycle pump contains air at 290K. The piston of the pump is slowly pushed in until the volume of the air pump. The outlet is then sealed off and the piston suddenly pulled out to full extension. If no air escapes. Find its temperature immediately after pulling the piston (take $\frac{C_P}{C_V} = 1.4$) **An[152.3K]**

CHAPTER4: KINETIC THEORY OF GASES

4.1: Brownian motion

It's a continuous random and haphazard motion of fluid particles caused by repeated collision of particles exerting a resulting force on each other which changes in a magnitudes and direction

Kinetic theory of matter states that Matter is made up of small particles called molecular atoms that are in continuous random motion and the speed of movement of the particles is directly proportional to temperature.

Explain why gas fills container in which it is placed and exerts pressure on the walls using kinetic theory of gases.

- A gas contains molecules with a negligible intermolecular forces and are free to move in all directions. As they move they collide with each other and with the walls of the container. The movement makes them fill the available space and the collisions with the walls constitute the pressure exerted on the wall

Explain using kinetic theory why the pressure of fixed mass of gas rises when its temperature is increased at constant volume.

- When gas temperature increases, the average kinetic energy of molecules increases, they make more frequent collisions with the walls of the container. This implies greater pressure of the gas. In addition pressure increases as a result of a higher rate of change of momentum at each collision.

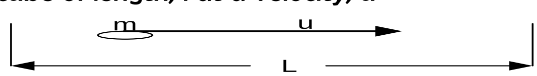
4.2: Derivation of expression of pressure exerted on container by the gas ($P = \frac{1}{3}\rho\overline{C^2}$)

In deriving this expression, the following assumptions are considered;

- Intermolecular forces of attraction are negligible
- Molecules make perfectly elastic collisions
- The volume of molecules is negligible compared to the volume of container.
- The duration of collision is negligible compared with time between collisions.

Derivation of expression $P = \frac{1}{3}\rho\overline{C^2}$

Consider a molecule of mass, m moving in a cube of length, l at a velocity, u



Change in momentum $= mu - (-mu) = 2mu$

Rate of change in momentum$= \frac{2mu}{t}$

But time, t between collisions$= \frac{2L}{u}$

Force on the wall by molecule, $F_1 = \frac{2mu_1}{\left(\frac{2L}{u_1}\right)} = \frac{mu^2{}_1}{l}$

For N molecules, force on the wall, F

$F = \frac{mu^2{}_1}{l} + \frac{mu^2{}_2}{l} + \dots\dots \frac{mu^2{}_N}{l}$

Pressure, $P = \frac{F}{A} = \frac{m}{l^3}(u_1^2 + u_2^2 + \cdots\cdots u_N^2)$

since $A = l^2$

but $\overline{U^2} = \frac{U_1^2 + U_2^2 \dots\dots\dots + U_N^2}{N}$

$\therefore N\,\overline{U^2} = U_1^2 + U_2^2 \dots\dots + U_N^2$

$P = \frac{Nm\overline{U^2}}{L^3} = \rho\overline{U^2}$ since $\rho = \frac{Nm}{L^3}$

The molecules do not show any preferences in moving parallel to any direction.

$\overline{C^2} = \overline{U^2} + \overline{V^2} + \overline{W^2}$ and $\overline{U^2} = \overline{V^2} = \overline{W^2}$

$\overline{C^2} = 3\overline{U^2}$

$\therefore \overline{U^2} = \frac{1}{3}\overline{C^2}$

$$\boxed{P = \frac{1}{3}\rho\overline{C^2}}$$

Since density, $\rho = \frac{Nm}{V}$ where m is mass of one molecule

$P = \frac{1}{3}\frac{Nm}{V}\overline{C^2}$

$$\boxed{PV = \frac{1}{3}Nm\overline{C^2}}$$

Examples

1) Given that density of oxygen is $0.098kgm^{-3}$ at a pressure of 1.0 x $10^5 Nm^{-2}$. Calculate the root mean square speed of oxygen

Solution

$$C_{r.m.s} = \sqrt{\frac{3\,P}{\rho}}$$

$$C_{r.m.s} = \sqrt{\frac{3\ x 1x10^5}{0.098}} = 1749.64ms^{-1}$$

2) Calculate the root mean square speed of molecule of an ideal gas at 130°C, given that the density of the gas at pressure of 1.0 x $10^5 Nm^{-2}$ and temperature of 0°C is 1.43kgm^{-3}

Solution

P_1 = 1.0 x 10^3, T_1 = 273K,

ρ = 1.43$kgm^{-3,}$

$C_{r.m.s}\ at\ 273K = ?$

T_2 = 403K, $C_{r.m.s}\ at\ 403K$

$P_1 = \frac{1}{3}\rho \overline{C_1^{\,2}}$

$$C_{r.m.s} = \sqrt{\frac{3\,P}{\rho}}$$

$$C_{r.m.s}\ at\ 273K = \sqrt{\frac{3x1.0x10^5}{1.43}}$$

$$C_{r.m.s}\ at\ 273K = \sqrt{209.79x10^3}$$

$$\frac{C_{r.m.s}\ at\ 273K}{C_{r.m.s}\ at\ 403K} = \frac{\sqrt{273}}{\sqrt{403}}$$

$$\frac{\sqrt{209.79x10^3}}{C_{r.m.s}\ at\ 403K} = \frac{\sqrt{273}}{\sqrt{403}}$$

$$C_{r.m.s}\ at\ 403K = 556.4878m/s$$

EXERCISE:44

1) The density of air is 1.3kgm^{-3}. Calculate the root mean square speed of air molecules in a container in which the pressure is 1.0 x $10^5 Nm^{-2}$.**An(480m/s)**
2) The density of nitrogen at s.t.p is 1.251kgm^{-3}. Calculate the root mean square velocity of nitrogen molecules at s.t.p **An(493m/s)**
3) The root mean square speed of helium at s.t.p is 1.3kgm^{-3}. Calculate the density of helium. **An(0.179 kgm^{-3})**
4) The density of nitrogen at s.t.p is 1.25kgm^{-3}. Calculate the root mean square velocity of nitrogen molecules at 227°C **An(666m/s)**
5) The root mean square speed of nitrogen at 127°C is 600m/s. Calculate the root mean square speed at 1127°C **An(1.12x10³m/s)**
6) Calculate the temperature at which the root mean square speed of oxygen molecules is twice as great as their root mean square speed at 27°C **An(1200K)**

4.4: Deductions of Dalton's law of partial pressures using kinetic theory

$$P = \frac{1}{3}\rho\overline{C^2}$$

Since density, ρ = $\frac{Nm}{V}$ where m is mass of one molecule

$$PV = \frac{1}{3}Nm\overline{C^2}$$

$$\therefore N = \frac{3PV}{m\overline{C^2}}$$

If the gas has two components 1 and 2

$N_1 = \frac{3P_1V}{m_1\overline{C^2}_1}$ and $N_2 = \frac{3P_2V}{m_2\overline{C^2}_2}$

N = $N_1 + N_2$

$$\frac{3PV}{m\overline{C^2}} = \frac{3P_1V}{m_1\overline{C^2}_1} + \frac{3P_2V}{m_2\overline{C^2}_2}$$

At constant temperature

½ m$\overline{C^2}$ = ½$m_1\overline{C^2}_1$ = ½ $m_2\overline{C^2}_2$

Hence P = P_1 + P_2

4.4: Real gases

Real gases obey ideal gas equation only when they are at very low pressure and at high temperatures.

Note: At high temperature and low pressure real gases behave like ideal gases.

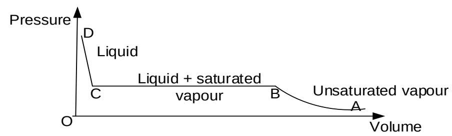
- At high temperature the average kinetic energy of the molecules is high and intermolecular separation increases, intermolecular forces are so weak such that they become negligible and thus the gas behaves like an ideal gas.
- At low pressure for a fixed number of molecules, volume increases. So the molecules will occupy a negligible volumes compared with that of the container. Hence the gas will behave like an ideal one

Properties of real gases

- Intermolecular forces of attraction and repulsion are not negligible

- ❖ Volumes of molecules are not negligible compared to volume of container
- ❖ The collision in real gases are inelastic
- ❖ They do not obey gas laws and equations

4.5: Pressure against volume curve for a real gas compressed below critical temperature



- ➢ In region AB, there is unsaturated vapour which fairly obeys Boyle's law at low pressures.
- ➢ At higher pressures (BC), some of the vapour condenses and we have liquid plus saturated vapour but pressure remains constant as volume reduces
- ➢ At much higher pressures (CD), all the vapour condenses into a liquid and there is a very small change in volume for a large pressure increase.

4.6: Vander-waal equation

Vander Waal modified the ideal gas equation by taking into account two of assumption made by kinetic theory to be valid.

Therefore Vander Waal's equation is given by $\boxed{\left(\mathbf{P} + \frac{\mathbf{a}}{\mathbf{V^2}}\right)(\mathbf{V} - \mathbf{b}) = \mathbf{nRT}}$

Accounting for the terms $\frac{a}{V^2}$ and b

- ❖ $\frac{a}{V^2}$ caters for <u>pressure defect,</u> since in real gases the <u>intermolecular forces</u> of attraction are <u>not negligible.</u> Therefore the observed pressure is actually less than the pressure in the ideal case by an amount
- ❖ The factor b accounts for <u>co-volume,</u> since the <u>volume</u> of the molecules of a real gas is <u>not negligible</u> compared to the volume of the gas.

4.7: Vapours

A vapour is gaseous state of substance below its critical temperature. A vapour can either be saturated or unsaturated

A gas is a gaseous state of substance above it's critical temperature

Supper saturated vapor is one whose rate of evaporation exceeds its rate of condensation.

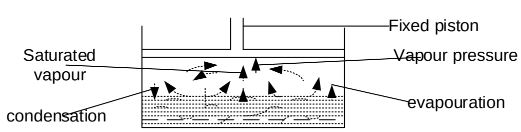
4.7.1: Saturated and Unsaturated Vapour

- ❖ A saturated vapour is one which is in dynamic equilibrium with it's own liquid. Saturated vapours do not obey gas laws
- ❖ Unsaturated vapour is one which is not in dynamic equilibrium with it's own liquid. Un saturated vapours approximately obey gas laws

4.7.2: Saturated vapour pressure (s.v.p)

S.V.P of a liquid is the maximum constant pressure exerted by the vapour in dynamic equilibrium with its liquid

(a): Explanation of occurrence of S.V.P using kinetic theory



- ➢ Consider a liquid confined in the container with fixed piston. The liquid molecules are moving randomly with <u>mean kinetic energy</u> determined by <u>liquid temperature</u>.

The most energetic molecules have sufficient K.e to overcome the attraction by other molecules and leave the surface of liquid to become vapour molecules by a process of **evaporation** .

- The molecules of the vapour are also moving randomly with a mean kinetic. The vapour molecule collide with walls of the vessel giving rise to vapour pressure and others bombard the surface of the liquid and re-enter the liquid by **condensation**.
- A state of dynamic equilibrium is attained when the rate of condensation equals to rate of evapouration. At this point the density of vapour and hence vapour pressure is maximum and constant at that temperature of the vapour and this is called S.V.P.

(b): Effect of volume change on S.V.P at constant temperature

- When the volume of saturated vapour is decreased at constant temperature, the density of vapour increases and the rate of condensation increases.
- As a result more molecules return to the liquid than leave it. The number of molecules in the vapour continue to fall until dynamic equilibrium is again restored with SVP having the **original value**.

(c) Effects of increasing temperature on SVP at constant volume

If a liquid is in dynamic equilibrium with its vapour, an increase in temperature increases the mean kinetic energy of molecules and hence evapouration rate increases. The vapour density increases, implying increase in the rate of condensation until a dynamic equilibrium is restored. There are now more molecules in the vapour phase than previously that are moving faster and hence higher pressure.

4.8: Experiment to verify variation of S.V.P with temperature

- Atmospheric pressure, H is obtained
- Tap is opened and pressure varied a vacuum pump to a suitable value
- The tap is closed and the liquid is heated until it boils.
- The temperature θ of the vapour is determined using a thermometer and noted.
- The difference, h in mercury levels is noted from the manometer.
- The saturated vapour pressure, $P = H \pm h$ is calculated
- The procedure is repeated for to obtain corresponding values of P and θ
- A graph of P against θ is plotted. The graph shows P increases with temperature.

4.9: Boiling

This is defined as the process by which a liquid turns to vapor at constant temperature (boiling point)

Boiling point of liquid is the constant temperature at which saturated vapour pressure is equal to external atmospheric pressure.

Explanation of boiling using kinetic theory

- Molecules of a liquid though moving randomly have attractive forces between them. When a liquid is heated molecules move faster and forces of attraction are weakened until they overcome at the boiling point temperature.

- At boiling point the saturated vapour pressure of the liquid is equal to the external pressure (atmospheric pressure plus hydrostatic pressure plus the pressure due to surface tension). The liquid molecules with enough energy escape from the bulk to the atmosphere

Effect of pressure on boiling point of a liquid

Increase of pressure raises the boiling point. Boiling takes place when SVP just exceeds external pressure. SVP increases with temperature so increase external pressure and therefore increase in boiling point

Effect of altitude on boiling point of a liquid

Boiling takes place when SVP just exceeds external pressure. Atmospheric pressure reduces with increase in altitude, therefore boiling point of a liquid decreases with increase in altitude,

Question: Explain why at a given external pressure a liquid boils at a constant temperature.

A liquid boils when saturated vapour pressure is equal to the external pressure. But since the saturated vapour pressure is dependent on the temp of the liquid, then it implies that for a given external pressure the boiling will occur at a constant temperature.

Question: Explain why the temperature of a liquid does not change when the liquid is boiling.

At boiling point, there is change in state to vapour and all the heat supplied is used to do work by breaking the molecular bonds of the liquid. The temperature will not change until all the bonds are broken

NB:

- Water can be made to boil at temperature less than 100 ℃ by boiling it at higher altitude or boiling it when it is free of impurities.
- Addition of impurities raise the boiling point of a liquid since impurities absorb some of the supplied heat making the liquid to boil at a higher temperature than its normal boiling point thus faster cooking.

4.10: Evapouration

This is the process by which a liquid become a vapour and leaves a liquid surface.
It can take place at all temperatures and only at the surface but it is greatest when the liquid is at it's boiling point.

Explanation using kinetic theory

- Evaporation occurs when the most energetic molecules at the liquid surface escape.
- The molecules that remain are those with low kinetic energy. Since mean kinetic energy of the molecules is directly proportional to absolute temperature, the liquid cools

Ways of increasing evapouration

- Increasing surface area of liquid
- Increasing temperature of the liquid
- Reducing air pressure above the liquid
- Causing a drought to remove vapour molecule before they have any chance to retain the liquid.

Differences between evapouration and boiling

- Boiling occurs through out the volume of the liquid while evapouration occurs at the surface.
- A liquid boils at single temp for any given external pressure whereas evaporation occurs at any temperature.

4.11: Melting

This is defined as the process by which a solid turns to liquid at constant temperature called melting point i.e.

Melting point is constant temperature at which a solid substance liquidizes at constant atmospheric pressure

Question: Explain why the temperature of a solid does not change when the solid is melting.
During melting, the heat energy supplied is used to weaken the intermolecular forces and increase separation between molecules. This increases the potential energy of the molecules but the mean kinetic energy of the molecules remain constant consequently the temperature remaining constant.

Question: Explain what happens when a fixed mass of ice is raised from $0°C$ to $10°C$
AT $0°C$ the bonds between ice are weakened and ice melts. Between $0°C$ and $4°C$ water contracts. Beyond $4°C$ water expands

Related explanations

- Metallic utensils being good conductors of heat, they absorb heat (from food) which would be carried away by the volatile liquid to the cooling fins thus delaying the refrigerating process. Such utensils are not recommended to be used in refrigerators.
- Milk in a bottle wrapped in a wet cloth cools faster than that placed in a bucket exposed to a drought. This is because the wet cloth speeds up the rate of evaporation thus more cooling.
- It advisable for a heavily perspiring person to stand in a shade other than drought because drought speeds up evaporation thus faster cooling which may lead to over cooling of the body and eventually this over cooling may lower the body's resistance to infections.
- When taking a bath using cold water, the individual feels colder on a very shiny day than on a rainy day because on a shiny day, the body is at high temperatures such that on pouring cold water on the body, water absorbs some of the body's heat thus its cooling. Yet on a rainy day the body is at a relatively low temperature implying that less heat is absorbed from it when cold water is poured on it.
- Two individuals; **A** (suffering from serious malaria) and **B** (normal) taking a bath of cold water at the same time of the day, **A** feels colder than **B** because the sick person's body is at relatively higher temperature than of a normal person. When cold water is poured on the sick person's body, much heat is absorbed from it compared to that absorbed from a normal person thus more coldness.
- Two normal identical individuals; **A** (takes a bath of water at 35 ℃) and **B** (takes a bath of water at 25 ℃) after the bath, **A** experience more coldness than **B**. This is because Water at 35 ℃ raises the body's temperature more than that at 25 ℃. This means that after the bath, the individual who takes a bath of water at 35 ℃ looses more heat to the surrounding than what one who takes a bath of water at 25 ℃ would loose to it.
- Water bottles are made of plastic other than glass and not fully filled because when water cools, it expands such that ice takes up a bigger volume. The un filled space is to cater for increase in volume on solidification and the bottle is made plastic to with stand breaking due to increase in volume
- A cloudy film forms on screens of cars being driven in rain because of the condensation of the excess water vapor in atmospheric moist air as a result of exceeding its dew point.

UNEB 2019 Q.6

(a) (i) What is meant by **a reversible process?** (02marks)

(ii) Distiguish between **saturated** vapour and **unsaturated** vapour (02marks)

(iii) Explain why evaporation causes cooling (03marks)

(b) Describe an experiment to determine the temperature dependence of saturated vapour pressure of water (07marks)

(c) (i) State **Daltons law of partial pressures** (01mark)

(ii) A sealed container liquid water, water vapour and air all at 27°C. the total pressure inside the container is 69cmHg. When the temperature is now raised to 85°C, the total pressure changes to 96cmHg. If the saturated pressure vapour of water at 27°C is 5cmHg, calculate the saturated vapour pressure of water at 85°C. **An[19.63cmHg]** (05marks)

UNEB 2017 Q.6

(a) (i) What is meant by **Boiling point** (01mark)

(ii) Explain why boiling point of a liquid increases with increase in the external pressure (04marks)

(b) (i) Explain how the pressure of a fixed mass of a gas can be increased at

- Constant temperature. (03marks)
- Constant volume. (03marks)

(a) (i) Sketch a pressure versus volume curve for a real gas undergoing compression. (02marks)

(ii) Explain the main features of the curve in (c)(i) above (03marks)

(b) The cylinder of an exhaust pump has a volume of $25cm^3$. If it is connected through a valve to a flask of volume $225cm^3$ containing air at a pressure of 75cmHg, calculate the pressure of the air I the flask after two strokes of the pump, assuming that the temperature of the air remains constant (04marks) **An(60.8cmHg)**

UNEB 2016 Q.6

(a) (i) State **Dalton's law of partial pressures** (01mark)

(ii) The kinetic theory expression for the pressure P, of an ideal gas of density ρ, and mean square speed, $\overline{c^2}$ is $\boldsymbol{P = \frac{1}{3}\rho \overline{C^2}}$. Use the expression to deduce Dalton's law (05marks)

(b) (i) What is meant by **isothermal** process and **adiabatic** process. (02marks)

(ii) Explain why a diabatic expansion of a gas causes cooling. (03marks)

(c) A gas at a temperature of 17°C and pressure of $1.0x10^5 Pa$ is compressed isothermally to half its original volume. It is then allowed to expand adiabatically to its original volume.

(i) Sketch on a P-V curve the above processes. (02marks)

(ii) If the specific heat capacity at constant pressure is 2100$Jmol^{-1}K^{-1}$ and at constant volume is 1500$Jmol^{-1}K^{-1}$, find the final temperature of the gas. **An(219.8K)** (04marks)

(d) (i) What is meant by **a saturated vapour** (01mark)

(ii) Explain briefly the effect of altitude on the boiling point of a liquid (02marks)

UNEB 2014 Q.5

(a) (i) State **two** differences between **saturated** and **unsaturated** vapours (02marks)

(ii) Sketch graphs of pressure against temperature for an ideal gas and for saturated water vapour originally at 0°C (03marks)

(b) The specific heat capacity of oxygen at constant volume is 719$Jkg^{-1}K^{-1}$ and its density at standard temperature and pressure is 1.49kgm^{-3}. Calculate the specific heat capacity of oxygen at a constant pressure $\boldsymbol{An(977.9Jkg^{-1}K^{-1})}$ (04marks)

(c) (i) With the aid of a labelled diagram, describe an experiment to determine saturated vapour pressure of water (05marks)

(ii) State how the experimental setup in (c) (i) may be modified to determine a saturated vapour pressure above atmospheric pressur3e (01mark)

(d) (i) Define an ideal gas (01mark)

(ii) State and explain the conditions under which real gases behave as ideal gas (04marks)

CHAPTER5: HEAT TRANSFER

There are 3 ways of heat transfer namely;

- Conduction
- Radiation
- Convection

5.1: Conduction

This is the process of heat transfer through a substance from region of high temperature to low temperature without the bulk movement of the molecules.
It is mainly due to collision between atoms that vibrate about their fixed positions

5.1.2: Mechanisms of heat conduction

a) In non metallic solids and fluids (poor conductors).

When one end of a poor conductor is heated, atoms at the hot end vibrate with increased amplitudes, collide with neigbouring atoms and lose energy to them. The neighbouring atoms also vibrate with increased amplitudes, collide with adjacent atoms and lose energy to them. In this way, heat energy is transmitted from one end to the other.

b) In metals (good conductors).

- Metals have free electrons. When heated the electrons at the hot end gain more energy and transfer energy as they collide with atoms in solid lattice.
- The mechanism of heat transfer by atomic vibrations also occurs in good conductors but its effect is much smaller

Question: Explain why metals are better conductor than non metallic solids.

In metals heat is carried by inter atomic vibration just like in non- metallic solid. But in addition to this, metals have free electrons in their lattice that move with very high velocity when heated since they are light. So they pass on their heat energy due to collision with the atoms in metallic lattice and this occurs at faster rate

5.1.3: Thermal conductivity or coefficient of thermal conductivity (K)

Thermal conductivity is the rate of heat flow through material per unit cross-sectional area per unit temperature gradient

S.I unit of K is $W\ m^{-1}K^{-1}$

Examples

1. An aluminum plate of cross section area 300cm^2 and thickness 5cm has one side maintained at 100°C by steam and another side by 30°C .The energy passes through the plate at a rate of 9kW.Calculate the coefficient of thermal conductivity of aluminum.

Solution

$$K = \frac{L\frac{Q}{t}}{A(\theta_2 - \theta_1)}$$

$$K = \frac{5x10^{-2}x9000}{300x10^{-4}x(100-30)} = 214.29\ Wm^{-1}K^{-1}$$

2. Calculate the quantity of heat conducted through $2m^2$ of a brick wall 12cm thick in 1 hour, if the temperature on one side is 18°C and on the other side is 28°C. Thermal conductivity of brick 0.13 $Wm^{-1}K^{-1}$

Solution

$$\frac{Q}{t} = \frac{K_b A(\Delta\theta)}{L_b}$$

$$Q = \frac{0.13x2(28-18)}{12x10^{-2}}x1x3600$$

$$Q = 7.8x10^4 J$$

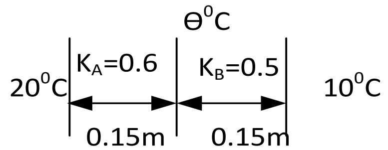
5.1.6: Heat flow through several surfaces

i) Surface in Series

Examples

1) wall 6m by 3m consists of two layers A and B of thermal conductivities $0.6\ Wm^{-1}K^{-1}$ and $0.5\ Wm^{-1}K^{-1}$ respectively. The thickness of layer is 15.0cm.The inner surface of layer A is at temperature of 20°C while outer layer B is at temperature of 10°C. Calculate
   (i) The temperature of interface of A and B. (ii) The rate of heat through wall

Solution



$$\frac{Q}{t} = \frac{K_aA(20-\theta)}{L_a} = \frac{K_bA(\theta-10)}{L_b}$$

$$\frac{0.6A(20-\theta)}{0.15} = \frac{0.5A(\theta-10)}{0.15}$$

$$\theta = 15.45°C$$

ii) $$\frac{Q}{t} = \frac{K_aA(20-\theta)}{L_a} = \frac{0.6A(20-\theta)}{15x10^{-2}}$$

$$\frac{Q}{t} = \frac{0.6x6x3(20-15.45)}{15x10^{-2}} = 324Js^{-1}$$

2) A sheet of rubber and a sheet of card board, each 2mm thick, are pressed together and their outer faces are maintained respectively at 0°C and 25°C. If the thermal conductivities of rubber and cardboard are respectively 0.13 and 0.05 $Wm^{-1}K^{-1}$, find the quantity of heat which flows in 1 hour across the composite sheet of area 100cm^2

Solution

$$\frac{Q}{t} = \frac{K_RA(\theta-0)}{L_R} = \frac{K_BA(25-\theta)}{L_B}$$

$$\frac{Q}{t} = \frac{0.13A(\theta-0)}{2x10^{-3}} = \frac{0.05A(25-\theta)}{2x10^{-3}}$$

$$\frac{0.13A(\theta-0)}{2x10^{-3}} = \frac{0.05A(25-\theta)}{2x10^{-3}}$$

$$\theta = 7°C$$

$$\frac{Q}{t} = \frac{0.13A(\theta-0)}{2x10^{-3}}$$

$$Q = \frac{0.13x100x10^{-4}(\theta-0)}{2x10^{-3}}x1x60x60$$

$$Q = 1.64x10^4J$$

3) A copper rod 2m long and of diameter 3cm is lagged. One end is maintained at 300°C, the other end is placed against 3cm thick card board disk of same diameter as the rod. The free end of disk is maintained at 40°C .Calculate;
   (i) Steady state temperature at copper card board junction.
   (ii) Quantity of heat flowing against junction in 10 minutes.

   (Thermal conductivity of copper and card board are 380 and 0. $2Wm^{-1}K^{-1}$ respectively).

Solution

$$\frac{Q}{t} = \frac{380x\left[\frac{\frac{22}{7}(3x10^{-2})^2}{4}\right](300-291.19)}{2} = 1.183Js^{-1}$$

$$Q = 1.183xt = 1.183x10x60 = 709.8$$

ii) Surface in parallel

1) A small green house consists of 34m^2 of glass of thickness 3.0mm and 9.0m^2 of concrete wall of thickness 0.080m. On a sunny day, the interior of the green house receives a steady 25kW of solar radiation. Estimate the difference in temperature between inside and outside of the green house. The temperature inside and outside may be assumed uniform and heat transfers downwards into the ground inside the green house may be neglected.(Thermal conductivity of glass and concrete are 0.85 $Wm^{-1}K^{-1}$, 1.5 $Wm^{-1}K^{-1}$ respectively)

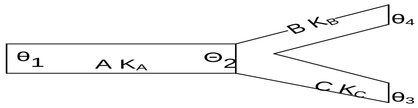
Solution

$$\frac{Q}{t} = \frac{K_GA(\theta_2-\theta_1)}{L_G} + \frac{K_CA(\theta_2-\theta_1)}{L_C}$$

$$25000 = \frac{0.85x34(\theta_2-\theta_1)}{0.003} + \frac{9x1.5(\theta_2-\theta_1)}{0.08}$$

$$\theta_2 - \theta_1 = 2.55°C$$

i) Surface not in series (Y shaped)



$$\frac{Q}{t} = \frac{K_A A(\theta_1 - \theta_2)}{L_A} = \frac{K_B A(\theta_{2-}\theta_4)}{L_B} + \frac{K_C A(\theta_{2-}\theta_3)}{L_C}$$

Example

Rods of copper, brass and steel are welded together to form Y-Shaped figure. The cross sectional area of each rod is 2cm^2 .The end of copper rod maintained at 100°C and the ends of brass and steel rod at 0°C, assume that there is not heat loss from surface of rod and that length of rods are 46cm, 13cm and 12cm respectively. Calculate the;

(i) temperature of junction. (ii) heat current in the copper rod

(thermal conductivities of copper, brass and steel are respectively 385 $Wm^{-1}K^{-1}$, 109 $Wm^{-1}K^{-1}$and 50.2 $Wm^{-1}K^{-1}$

Solution

$$\frac{Q}{t} = \frac{K_C A(100-\theta)}{L_C} = \frac{K_B A(\theta-0)}{L_B} + \frac{K_S A(\theta-0)}{L_S}$$

$$\frac{385(100-\theta)}{0.46} = \frac{109(\theta-0)}{0.13} + \frac{50.2(\theta-0)}{0.12}$$

$$8369565 - 836.9565\,\theta = 418.33\,\theta + 838.46\,\theta$$

$$\theta = \mathbf{39.97}℃$$

$$\frac{Q}{t} = \frac{K_C A(100-\theta)}{L_C} = \frac{3852x10^{-4}(100-39.97)}{0.46} = \mathbf{10.05\ Js^{-1}}$$

5.1.7: Relationship between rate of heat flow and latent heat of vapourisation.

$$\frac{Q}{t} = ML.$$

Where $M =$ Mass per unit time

$L =$ Latent heat of Vapourisation

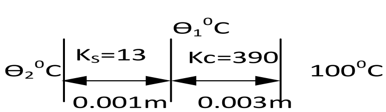
Examples

1) An Iron saucepan containing water which boils steadily at 100°C stands on a hot plate and heat is conducted through the base of the pan of area 4m^2 and uniform thickness 2 x 10^{-3}m. If water evaporate at a rate of 0.09 kg/min. Calculate the surface temperature of out side surface of the pan.(Thermal conductivity of Iron = 66 $Wm^{-1}K^{-1}$ and Lv = 2.2 x 10^6 JK^{-1})

Solution

2) A copper kettle has Circular base of radius 10cm and thickness 3mm, the upper surface of base is covered with a uniform layer of soot 1mm thick. Kettle contains water which is boiled to boiling point by an electrical heat. In steady state 5g of steam are produced each minute. What is the temperature of the lower surface of the base assuming that heat conduction from the side of the kettle can be ignored (thermal conductivity of copper and soot respectively are 390 $Wm^{-1}K^{-1}$ and 13.0 $Wm^{-1}K^{-1}$ and $L_v =$ 2.26 x 10^{6}Jkg^{-1} .)

Solution



$$\frac{Q}{t} = ML = \frac{5x10^{-3}}{60}x\,2.26\,x\,10^6 = 188.333$$

$$\frac{Q}{t} = \frac{K_s A(\theta_2 - \theta_1)}{L_S} = \frac{K_c A(\theta_{1-}100)}{L_C}$$

$$188.333 = \frac{13\pi x\left(10x10^{-2}\right)^2(\theta_2-\theta_1)}{1x10^{-3}}$$

$$0.188333 = \mathbf{0.4084}\theta_2 - 0.4084\theta_1 \ldots\ldots..1$$

Also: $188.333 = \frac{390\pi x\left(10x10^{-2}\right)^2(\theta_1-100)}{3x10^{-3}}$

$$0.564999 = \mathbf{12.2522}\theta_1 - 12.2522x100.\ldots.2$$

$$\theta_1 = 100.46℃$$

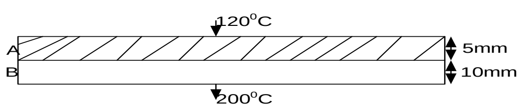
Put into (1);

$$0.1883 = \mathbf{0.4084}\theta_2 - 0.4084x100.46$$

$$\theta_2 = 105.06℃C$$

EXERCISE: 46

1) A well lagged composite metal bar of uniform cross section area 2cm^2 is made by joining 40cm rod of copper to 25cm rod of Aluminium. The extreme ends of the bar are maintained respectively at 100° C and 0°C respectively. Calculate;
   (i) The temperature of junction of two rods.
   (ii) Rate of heat flow
   (Thermal conductivity of copper and Aluminum is 386 and 210 $Wm^{-1}K^{-1}$ respectively).
   An (i) 53.5℃ (ii) 8.9745J
2) A concrete floor of a hall has dimensions of 10.0m by 8.0m.It is covered with carpet of thickness 2.0cm.The temperature inside the hall is 22°C while that of the surrounding just below the concrete is 12°C.Thermal conductivity of concrete and carpet are 1 and 0.05 $Wm^{-1}K^{-1}$respectively and thickness of concrete is 10cm. Calculate
   (i) Temperature at the interface of concrete and Carpet
   (ii) The rate at which flow through the floor.
   An(14° C , 1600W)
3)



The metal conductors A and B each of radius 20cm and thickness 5mm and 10mm respectively are placed in contact as shown above. The upper surface of A and lower surface of B are maintained at temperature of 120°C and 200° C respectively. Calculate;
   (i) Temperature of interface
   (ii) Rate of hard flow through A
   An(138.9°C, 99.75 x 10^{3}w)
   (Thermal conductivities of A and B are 210 and 130 $Wm^{-1}K^{-1}$respectively)
4) Ice is forming on the surface of a pond. When it is 4.6cm thick, the temperature of the surface of the ice in contact with air is 260K, while the surface in contact with the water is at temperature 273K. calculate the;
   (i) rate of heat per unit are from the water
   (ii) Rate at which the thickness of the ice is increasing
   (if the thermal conductivity of the ice is $2.3\ Wm^{-1}K^{-1}$ and specific latent heat of fusion of ice is $3.25x10^5Jkg^{-1}$, $\rho\ of\ \mathrm{H_2O} = 1000kgm^{-3}$).
   An(6.5$x10^2Wm^{-2}, 2.0x10^{-3}mms^{-1}$)

5.1.8: Determination of thermal conductivity K

(a) Determination of thermal conductivity K of a good conductor of heat e.g copper using searle's method

Searle's method is best suited for a good conductor because it achieves measurable temperature gradient and measurable heat flow and this can be obtained by good conductor.

- A long copper bar of cross-sectional area A is used.
- It carries a heater at one end and copper coil soldered at the other end.
- Two thermometers are inserted in the holes drilled in the bar at a known separation l
- The holes are smeared with mercury for good thermal contact
- Water is allowed to flow through the copper coil and the heater is switched on.
- When the thermometers read steady temperatures θ_1, θ_2, θ_3 and θ_4 , recorded from thermometers T_1, T_2, T_3 and T_4 respectively.

$\frac{Q}{t} = \frac{KA(\theta_2-\theta_1)}{l}$ where k is thermal conductivity of copper metal

- The mass m of water flowing out per second through the coil is measured and recorded.

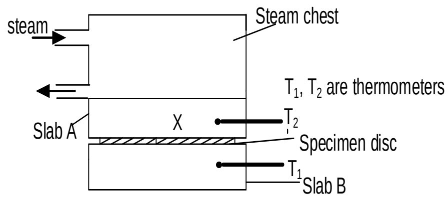
$\frac{Q}{t} = mc(\theta_4 - \theta_3)$ where c is specific heat capacity of water

- Therefore thermal conductivity, k of a good conductor is got from

$$\boxed{K = \frac{MCL(\theta_4-\theta_3)}{A(\theta_2-\theta_1)}}$$

(b) Determination of thermal conductivity (K) of a poor conductor e.g rubber,cork, glass using chest or lee disk method.

For a poor conductor, the material has to be made thin so that a measurable temperature gradient can be obtained and an adequate heat flow



❖ A sample in the form of a disc of small <u>thickness</u> l and <u>diameter</u>, D is used.

❖ The mass, m of slab B of specific heat capacity, c is determined.

$$\frac{Q}{t} = mcs$$

❖ Thermal conductivity, k of the disc is got from

$$mcs = \frac{k\ \pi d^2}{4}\frac{(\theta_2 - \theta_1)}{l}$$

5.2: RADIATION

Thermal radiation is a means of heat flow from hot places to cold places by means of electromagnetic waves.

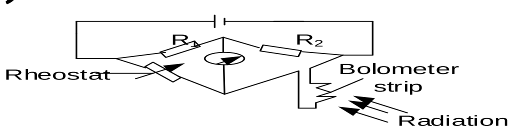
Radiation emitted by a hot body is a mixture of different wavelength. The amount of radiation for a given wavelength depends on the temperature of the body. At lower temperature, the body emits mainly infrared and at high temperatures the body emits ultraviolet, visible in addition to infrared

5.2.2:Properties of infrared radiation(electromagnetic radiations)

- ❖ Move at a speed of light ($3\text{x}10^8 ms^{-1}$)
- ❖ It can be reflected and refracted just like light
- ❖ Cause an increase in temperature when absorbed by matter
- ❖ It can cause photo electric emission surface
- ❖ It affects special types of photographic plates and it enables pictures to be taken in dark
- ❖ It is absorbed by glass but is transmitted by rock salt and quartz

5.2.3:Detection of infrared radiations

(a) Bolometer

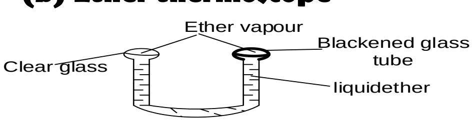


❖ A bolometer is connected to a wheatstone bridge circuit and its resistance measured.

❖ The radiation is allowed to fall on the bolometer which is then absorbed and the temperature increases

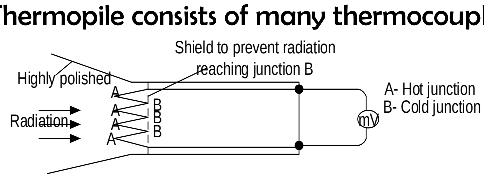
❖ The new resistance of the bolometer is also measured. An increase in resistance obtained detects infrared radiations

(b) Ether thermoscope



(c) Thermopile

Thermopile consists of many thermocouples connected in series



➢ Radiation falling on junction A is absorbed and temperature rises above that of junction B.

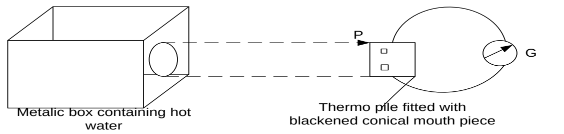
➢ An $E.m.f$ is generated and is measured by mill voltmeter connected directly to the thermopile and deflects as a result.

5.2.4: PREVOST'S THEORY OF HEAT EXCHANGE

Its states that, when a body is in thermodynamic equilibrium with its surrounding, its rate of emission of radiations to the surrounding is equal to its rate of absorption of radiations from the surrounding.

It is concluded in provosts theory that a good absorber of radiation, must also be a good emitter otherwise its temperature would rise above that of its surrounding.

5.2.5: EXPERIMENT TO DETERMINE WHICH SURFACE ARE GOOD ABSORBERS AND POOR ABSOBERS OF HEAT RADIATION

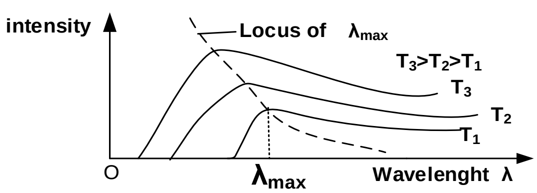


- A metal cube whose sides have a variety of finishes dull black, white highly polished is used
- The metal cube is filled with water and water is kept boiling at by a constant supply of heat
- A thermopile is made to face the various finishes of the cube at equal distances and each time the deflection on the galvanometer noted.
- The galvanometer deflection is greatest when the thermopile faces the dull black surface and less when it is facing the highly polished surface
- This means that a highly polished surface is a poor radiator and the dull black surface is the better radiator.

5.2.6: BLACK BODY RADIATION

A black body radiation is the radiation whose quality (wave length) depends only on the temperature of the body.

Spectral curves for black body radiation



- As the temperature increases, the intensity for every wavelength increases but the intensity for a shorter wavelength increases more rapidly
- At each temperature, there is a maximum intensity for a particular wavelength.
- λ_{max} decreases as temperature increases

Wein's displacement law

It states that the wavelength λ_{max}, for which the radiation emitted by a black body has maximum intensity is inversely proportional to the absolute temperature of the body

i.e $\lambda_{max} \alpha \frac{1}{T}$ or $\boxed{\lambda\text{max T} = \text{constant}}$

wein's displacement constant $= 2.9x10^{-3}mK$

Examples

(i) Calculate the wavelength of the radiation emitted by a black body at $15x10^6$K

Solution

$\lambda\text{max T} = 2.9x10^{-3}$ | $\lambda\text{max} = \frac{2.9x10^{-3}}{15x10^6}$ | $\lambda\text{max} = 1.93\text{x}10^{-10}\text{m}$

- in the middle spectrum visible) and eventually to blue hot (λ_{max} in blue region)

Why center of fire appears white

This is because temperature is highest at the center of the fire and this corresponds to the energy intensity where all wavelength radiations are emitted. The combination of all the colours at this temperature makes the fire appear white

Question. State black body radiation laws. (Weins displacement law and Stefan-Boltzmann's law)

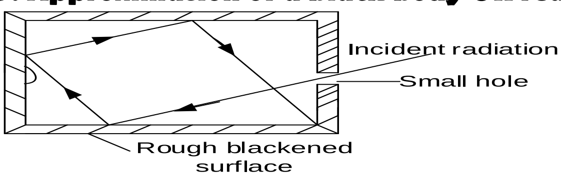
5.2.8:BLACK BODY

A black body is one which absorbs all radiations of every wavelength falling on it, reflects and transmits none.

Examples

- Furnace
- Star
- Sun

5.2.9: Approximation of a black body OR realization of black body



- ❖ A small hole is punched in a tin which is blackened inside.
- ❖ When a radiation is incident through the hole, it undergoes multiple reflections
- ❖ At each reflection energy is lost due to many reflections and all energy is lost reflections.

5.3: STEFAN'S LAW (STEFAN- BOLTZMAN'S LAW)

- ❖ It states that "the total power radiated per unit surface area of a black body is directly proportional to the fourth power of its absolute temperature" ($P\alpha T^4$)

OR

- ❖ Total energy radiated by a blackbody per unit surface area per unit time is directly proportional to the fourth power of its absolute temperature. ($E\alpha T^4$)

5.3.1:Expression for power radiated by black body

From Stefan's law

$\frac{energy}{surface\ area\ x\ Time} = \sigma T^4$ | $\frac{I.Vt}{S.t} = \sigma T^4$ | $\boxed{P = S\ \sigma T^4}$

Examples

1) A cylinder has radius 10^{-2}m and height $0.75mm$. Calculate the temperature of cylinder if it is assumed to be lamp of power 1kW. $\boldsymbol{\sigma} = 5.67x10^{-8}\boldsymbol{Wm^{-2}k^{-4}}$

Solution

$P = S\ \sigma T^4$ $\quad S = 2\pi rh$

$1000 = 5.67x10^{-8}x2\pi x10^{-2}x0.75x10^{-3}xT^4$

$T^4 = (3.74262x10^{14})$

$T = (3.74262x10^{14})^{\frac{1}{4}}$

$\text{T} = 4398.435\text{K}$

2) A cylindrical bulb filament of length $0.5m$ and radius $1.0x\ 10^{-4}m$ emits light as black body. $0.4A$ melts the filament when connected across 240V. Calculate;

(i) The melting point of the filament

(ii) The wave length of the radiation emitted at maximum intensity/emission at its melting point.

Solutions

i) $P = IV = S\ \sigma T^4$ $\quad S = 2\pi rh$

$0.4x240 = 5.67x10^{-8}x2\pi x1.0x10^{-4}x0.5xT^4$

$T^4 = (5.3894x10^{12})$

$T = (5.3894x10^{12})^{\frac{1}{4}}$

$\text{T} = 1523.648\text{K}$

ii) $\lambda\text{max T} = 2.9x10^{-3}$

$\lambda\text{max} = \frac{2.9x10^{-3}}{1523.648}$

$\lambda\text{max} = 1.90\mu\text{m}$

5.3.2: Expression for net power for a body in the surrounding

If a black body of surface area S is at absolute temperature T_0 placed in an environment which is at lower temperature T.

$$P_{net} = \boldsymbol{S\,\sigma}T_0^4 - \boldsymbol{S\,\sigma}T^4$$

$$\boxed{P_{net} = S\,\sigma(T_0^4 - T^4)} \text{ For } T_0 > T$$

Examples

1) Calculate the net loss of heat energy from space craft of surface area $25m^2$ and temperature of 300K if the radiation that it receives form the sun is equivalent to at temperature in the space 50K. Assume that the space craft behaves as a perfect black body.

$P_{net} = S\,\sigma(T_0^4 - T^4)$ | $P = 25x5.67x10^{-8}x(T_0^4 - T^4)$ | $P = 1.15x10^4W$

2) A small blackened solid copper sphere of radius 2cm is placed in evacuated enclosure those walls are kept at 100°C. find the rate at which energy must be supplied to sphere to keep its temperature constant at 127°C.

Solution

T_0=100°C $= 373K$, T =127°C $= 400K$

$P_{net} = S\,\sigma(T_0^4 - T^4)$ and $S = 4\,\pi r^2$

$P_{net} = 4\pi x(2x10^{-2})^2x5.67x10^{-8}(400^4-373^4)$

$P_{net} = 1.779W$

Note

If the body is not a black body, then the energy it emits at any temperature will be less than that emitted by a black body of similar surface area at the same temperature. The emission equation is modified as;

$$\boxed{P = eS\,\sigma T^4} \text{ where } e - emissivity$$

$$\boxed{P_{net} = e\,S\,\sigma(T_0^4 - T^4)} \text{ For } T_0 > T$$

Emissivity (e):
is defined as the ratio of total power emitted per squared meter of a given body to that emitted per squared meter of a black body at the same temperature as the body.

Examples

1. A 100W electric bulb has a filament which is 0.60m long and has a diameter of $8.0x10^{-5}m$. estimate the working temperature of the filament if its total emissivity is 0.70.

Solution

$P = eS\,\sigma T^4$ and $S = 2\pi rh$

$$100 = 0.70x2\pi x4x10^{-5}x0.6x5.67x10^{-8}xT^4$$

$$T = 2.02x10^3K$$

2. The surface area of a domestic hot water radiator made of iron 2mm thick is $4m^2$. If the water in the pipes is maintained at $60°C$ and the temperature of the room is $20°C$, calculate the quantity of heat supplied to the room per hour. (Assume the emissivity of the radiator surface is 0.4). **An**$(2.3x10^{10}J)$

5.3.3: SOLAR POWER / SOLAR CONSTANT

A solar power is the amount of energy received from the sun per second per meter squared.

Expression for solar constant

$$P_s = S\sigma T_s^4$$

Where S is its surface area of sun $(4\pi r_s^2)$

r_s is The radius of the sun

$power\ of\ the\ sun, P_s = 4\pi r_s^2\sigma T_s^4$

$$\boldsymbol{Solar\ power} = \frac{power\ of\ the\ sun}{surface\ area\ of\ the\ earth}$$

$$\boxed{\boldsymbol{Solar\ power} = \frac{\boldsymbol{4\pi r_s^2\sigma T_s^4}}{\boldsymbol{4\pi R^2}}}$$

Examples

1) The energy intensity received by a spherical planet from star is 1.4 x10^3Wm^{-2}. The star is of radius 7.0x10^5km and 14.0x10^7km from the planet. Calculate the surface temperature of star and state any assumptions made.

Solution

Solar power $= \frac{4\pi r_s^2 \sigma T_s^4}{4\pi R^2}$

1.4x10^3 $= \frac{(7.0x10^5x1000)^2 x5.67x10^{-8}T_s^4}{(14.0x10^7x1000)^2}$

$T = 5605.976K$

Assumption

-The star behaves as a black body
-The star is a perfect sphere
-All heat exchanges are by radiation

2) The flux of solar energy incident on the earth surface is $1.36\ x$$10^3$Wm^{-2}. If the sun's radius is 7.0x10^8m. It's distance from the earth is 1.52x10^{11}m. (speed of light=$3.0x10^8 ms^{-1}$, $\sigma = 5.67x10^{-8}Wm^{-2}K^{-4}$).
Calculate;
   (i) temperature of the surface of the sun
   (ii) total power emitted by the sun
   (iii) rate of loss of the mass by the sun

Solution

(i) Solar power $= \frac{4\pi r_s^2 \sigma T_s^4}{4\pi R^2}$

1400 $= \frac{(7.0x10^8)^2 x5.67x10^{-8}xT_s^4}{(1.52x10^{11})^2}$

$T_s = 5841.3K$

(ii) $P_s = 4\pi r_s^2 \sigma T_s^4$

$P_s = 4\pi x(7.0x10^8)^2 x5.67x10^{-8}(5841.3)^4$

power $= 4.06x10^{26}$W

(iii) $E = mc^2$

$Pt = mc^2$

$$\frac{m}{t} = \frac{4.06x10^{26}}{(3.0x10^8)^2} = 4.5x10^9 kgs^{-1}$$

5.3.4: EQUILIBRIUM OF THE SUN AND THE EARTH

The power radiated by the sun is given by
$P_s = 4\pi r_s^2 \sigma T_s^4$
Where $T_s =$ surface temperature of sun ,
$r_s =$ radius of sun

The solar power $= \frac{4\pi r_s^2 \sigma T_s^4}{4\pi R^2}$

The power received by the earth =solar power x area of earth
= solar power x πr_E^2
Where r_E – radius of earth

power received by the earth $= \frac{4\pi r_s^2 \sigma T_s^4}{4\pi R^2}$ x πr_E^2-- [1]

Earth also behaves like a black body, then the power radiated by the earth is

$P_E = 4\pi r_E^2 \sigma T_E^4$ -------------------------- [2]

$4\pi r_E^2 \sigma T_E^4 = \frac{4\pi r_s^2 \sigma T_s^4}{4\pi R^2}$ x πr_E^2

$T_E^4 = \frac{r_s^2}{4R^2} T_s^4$

$$\boxed{T_E^4 = \left(\frac{r_s}{2R}\right)^2 T_s^4}$$

Examples

1) Estimate the temperature of surface of earth if its distance from the sun 1.5x10^{11}m. Assume that the sun is sphere of radius 7.0x10^8m at temperatures 6000k

Solution

Solar power $= \frac{4\pi r_s^2 \sigma T_s^4}{4\pi R^2}$

Power received by earth $= \frac{4\pi r_s^2 \sigma T_s^4}{4\pi R^2}$x πr_e^2

Power radiated by earth= $4\pi r_e^2 \sigma T_e^4$

at equilibrium: Power radiated =power received

$4\pi r_e^2 \sigma T_e^4 = \frac{4\pi r_s^2 \sigma T_s^4}{4\pi R^2}$x πr_e^2

$T_e^4 = \frac{r_s^2}{4R^2} T_s^4$

$T_e = \left\{\frac{(7x10^8)^2 x6000^4}{4(1.5x10^{11})^2}\right\}^{\frac{1}{4}}$

$T_e = 290K$

2) Assume that the sun is sphere of radius 7.0x10^8m at temperatures 6000k. Estimate the temperature of surface of mars if its distance from the sun 2.28x10^{11}m.

Solution

Solar power = $\frac{4\pi r_s^2 \sigma T_s^4}{4\pi R^2}$

Power received by mars = $\frac{4\pi r_s^2 \sigma T_s^4}{4\pi R^2}$x $\pi r^2{}_m$

Power radiated by mars =$4\pi r_m^2 \sigma T_m^4$

At equilibrium: Power radiated =power received

$$4\pi r_m^2 \sigma T_m^4 = \frac{4\pi r_s^2 \sigma T_s^4}{4\pi R^2} \text{x } \pi r_m^2$$

$$T_m^4 = \frac{r_s^2}{4R^2} T_s^4$$

$$T_m = \left\{\frac{(7x10^8)^2 x6000^4}{4(2.28x10^{11})^2}\right\}^{\frac{1}{4}} = 235.08K$$

Exercise:49

1) The element of an electric fire, with an output of 1.0kW, is a cylinder 25cm long and 1.5cm in diameter. Calculate its temperature when in use , if it behaves as a blackbody.
(Stefan constant = $5.7x10^{-8}Wm^{-2}K^{-4}$) **An(1105K)**
2) Estimate the surface temperature of the earth assuming that it is radioactive equilibrium with the sun. (radius of sun $7.0x10^8$m, surface temperature of sun 6000K, distance from the earth to the sun $1.5x10^{11}$m, σ =$5.7x10^{-8}Wm^{-2}K^{-4}$) **An [289K]**
3) The normal operating condition of a variable-intensity car head lamp is 2.5A and 12V. The temperature of the filament is 1750℃. The intensity is now altered so that the lamp runs at 2.2A and 12.5V. Calculate the new operating temperature assuming that the filament behaves as a black body **An [1706K]**
4) A black body radiates heat at 2Wm^{-2} when at 0℃. Find the rate of fall in temperate of a copper sphere of radius 3cm when at 1000℃ in air at 0℃. (assume that the density of copper is 8930$kgm^{-3}$and its specific heat capacity is 385 $Jkg^{-1}K^{-1}$) **An [4.34Ks^{-1}]**
5) A certain 200W tungsten filament lamp operates at a temperature of 1500℃. Assuming that it behaves as a perfect black body estimate the surface area of the filament **An [3.56cm^2]**
6) Find the net rate of energy lost by radiation form the following black bodies
   (a) A sphere of radius 10cm at a temperature of 500℃ in an enclosure whose temperature is 20℃ **An [2505W]**
   (b) A person of surface area 1.2m^2 at a temperature of 37℃ in an enclosure whose temperature is 0℃. Comment on your answer **An [251W]**
7) A black body at 1000K emits radiation with maximum energy emitted at a wavelength of 25000nm. Calculate the wavelength at which maximum energy is emitted by the following assuming that they all behave as black bodies
   (a) A piece of iron heated in a Bunsen flame to 800℃ **An [3125nm]**
   (b) A star with a surface temperature of 7000℃ **An [357nm]**
   (c) The plasma in a fusion reaction at 10^6℃ **An [2.5pm]**

5.3.6:GREEN HOUSE EFFECT

- Short wavelength radiation from the sun passes through the atmosphere and is absorbed by plants and sand leading to higher earth temperature.
- Earth re-radiates long wavelength which is trapped by green house gases. Continued accumulation of this radiation implies higher earth temperature and with time may lead to global warming.

5.3.7: THERMAL CONVECTION

Is a process of heat transfer through a fluid from high temperature to low temperature due to actual movement of medium.
Heated fluid becomes less dense and is replaced by more dense fluid.

Mechanism of convection

When a fluid is heated underneath, it expands and becomes less dense than the fluid above. The warm less dense fluid rises to the top and the cooler more dense from above moves downwards to take its place. The circulating current of the fluid heats up the whole fluid

5.3.8:SEA BREEZE

During the day land heats faster than the sea, hot air expands and rises from the land. Cool air from the sea blows towards the land to replace up rising air, hence sea breeze occurs

5.3.9:LAND BREEZE

At night land cools faster than the sea, the sea still retains its warmth. warm less dense air from the sea rises and cool air from the land replaces it, hence land breeze occurs

Explain why cloudy nights are warmer than cloudless nights

During day, earth absorbs heat from sun. at night earth radiates heat into atmosphere. On cloudy night clouds reflect heat back to the earth and it feels warm. On cloudless night radiated heat is lost to atmosphere and earth feels colder

UNEB 2020 Q.7

(b) **(i)** Two cylindrical bodies A and B made of the same material but the length of A is twice that of B and the cross sectional area of B is a third that of A. If the ends of A and B are subjected to the same temperature difference, find the ratio of the rates of heat flow through A to the rate of heat flow through B. **An (3:2)** (03marks)

(ii) In the determination of thermal conductivity of copper, when water flows round the cool end of a copper rod at a rate of 600cm^3 per minute, its temperature increases by 3.3°C. The temperature at two points, a distance 5.2cm apart, along the copper rod are 70°C and 30°C repectively. Find the thermal conductivity of copper if the radius of the rod is 1.2cm. **An (398.5Wm^{-1}K^{-1})** (04marks)

(c) Describe an experiment to measure thermal conductivity of cork (07marks)

UNEB 2019 Q.5

(a) (i) State any **three** properties of ultraviolet radiation (03marks)

(ii) What is **a black body?.** (01mark)

(b) A cylindrical metal rod with a well insulated surface has one end blackened and then exposed to thermal radiation from a body at a temperature 500K. If the equilibrium temperature of the blackened end is 400K and the length of rod is 10m, calculate the temperature of the other end (thermal conductivity of the metal $= 500Wm^{-1}K^{-1}$), **An(358.11K)** (04marks)

(d) Explain why cloudy nights are warmer than cloudless nights (04marks)

UNEB 2019 Q.7

(a) Define the following

(i) Thermal conductivity (01mark)

(ii) Specific latent heat of vaporization (01mark)

(b) A boiler with a base made of steel 15cm thick, rests on a hot stove, the area of the bottom of the boiler is $1.5x10^3cm^2$. The water inside the boiler is at 100°C. If 750g of water is eveaporated every 5 minutes, find the temperature of the surface of the boiler in contact with the stove (thermal conductivity of the steel $= 50.2Wm^{-1}K^{-1}$, specific latent heat of vapouriastion of water$=$ $2.2x10^6JKg^{-1}$), **An(212.8°C)** (04marks)

(c) Hot water in a metal tank is kept constant at 65°C by an immersion heater in the water. The tank has a lagging all round it of thickness 20mm and thermal conductivity $0.04Wm^{-1}K^{-1}$ and its surface area is $0.5m^2$. The heat lost per second by the lagging is 0.8W per degree excess above the surroundings. Calculate the power of the immersion heater if the temperature of surrounding is 15°C

An(22W) (05marks)

UNEB 2018 Q.6

(a) (i) What is meant by **Conduction of heat?** (01mark)

(ii) Explain why mercury conducts heat better than water. (03marks)

(iii) Explain the occurrence of land and sea breeze. (06marks)

(b) A copper sphere of radius 7 cm and density $900kgm^{-3}$, is heated to a temperature of 127°C and then transferred to an evacuated enclosure whose walls are at a temperature of 27°C. Calculate the;

(i) Net rate of loss of heat by the coper sphere. (04marks)

(ii) Temperature of the copper sphere after 5 minutes. (04marks)

An$(61.122\,Js^{-1}, 109.3°C)$

(c) Explain why heating systems based on the circulation of steam are more efficient than those based on the circulation of boiling water. (02marks)

UNEB 2018 Q.7

(a) (i) What is meant by **a black body** (01mark)

(ii) Give **two** examples of a black body. (01mark)

(b) With the aid of graphs, describe how radiation emitted by a black body varies with wavelength for two temperatures. (05marks)

(c) (i) Define **thermal conductivity.** (01mark)

(ii) Describe an experiment to determine the thermal conductivity of glass. (07marks)

(d) Radiation from the sun falls normally on a blackened roof measuring $20m\ x\ 50\ m$. If half of the solar energy is lost in passing through the earth's atmosphere, calculate the energy incident on the roof per minute. (Temperature of the sun's surface $=\ 6000K$, radius of the sun$=\ 7.5x10^8m$, distance of the sun from the earth$=\ 1.5x10^{11}m$). (05marks)

Solution

Power radiated by sun, $P = 4\pi r^2\sigma T^4$

Power incident on sphere, $P_1 = \frac{P}{2} = 2\pi r^2\sigma T^4$

Power received by roof, $P_R = \frac{A_1}{A_2}P_1$

$$= \frac{20x50}{4\pi R^2}x2\pi r^2\sigma T^4$$

$$= \frac{1000}{2(1.5x10^{11})^2}x(7.5x10^8)^2\text{x}5.7x10^{-8}x6000^4$$

$$P_R = 923{,}400W$$

Energy incident on roof per minute, P_2

$$P_2 = 923{,}400x60 = 5.54x10^7\ J$$

UNEB 2017 Q.7

(a) (i) Define **thermal conductivity** (01mark)

(ii) Explain the mechanism of heat transfer by convection. (03marks)

(b) **(i)** State **Newton's law of cooling.** (01marks)

(ii) Describe briefly an experiment to verify Newton's law of cooling. (05marks)

(c) A wall is constructed with two types of bricks. The temperature of inner and outer surfaces of the wall are 29°C and 21°C respectively . The value of the thermal conductivity for the inner brick is $0.4Wm^{-1}K^{-1}$ and that of the outer brick is $0.8Wm^{-1}K^{-1}$

(i) Explain why in steady state, the rate of thermal energy transfer must be the same in both layers (02marks)

(ii) Calculate the temperature at the interface between the layers, if each layer is 12.0cm thick

An$(23.7°C)$ (04marks)

(d) Explain the green house effect and how it leads to rise of the earth temperature. (04marks)

SECTIONC: MODERN PHYSICS

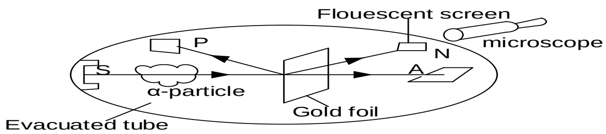
CHAPTER 1: ATOMIC STRUCTURE

An atom is a neutral particle made of central positive charge (nucleus) with negative charges (electrons) orbiting around it.

1.1: RUTHERFORD'S MODEL OF THE ATOM

Rutherford's model states: that the positive charge of the atom and nearly all its mass is concentrated in a very small volume at the centre with electrons in motion in a circular orbit around the nucleus.

RUTHERFORD'S ALPHA PARTICLE SCATTERING EXPERIMENT



- Alpha particles from a radioactive source were allowed to strike a thin gold foil placed in the centre of an evacuated vessel and the scattering of alpha particles when they collide with the gold foil was observed from a fluorescent screen mounted on a focal plane of a microscope.
- Alpha particles produce tiny, but a visible flash of light when they strike a fluorescent screen.
- Surprisingly, alpha particles not only struck the screen at A but also at N and some were even found to be back scattered to P.
- The greatest flash was observed at position A.

OBSERVATIONS

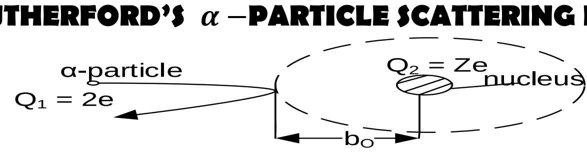
- **Most** of the alpha particle went through the gold foil **un deflected** . This is because the atom of the foil contains <u>very tiny nuclei</u> and most of the space of an atom is an <u>empty space</u>.
- **Few** alpha particles were scattered through small angles. This is because of the positive charge (nucleus) that strongly repelled the alpha particles
- **Very few** alpha particles were scattered through angles **greater than** $90°$. This is because positive charge (nucleus) occupies a very small volume of the atom, making the chance of head on collision very small

Failure of Rutherford's model of the atom

(1) An orbiting electron is constantly changing it's direction and therefore has an acceleration. In classical physics charges undergoing acceleration emit electromagnetic radiation continuously and therefore they would loose energy. This implies that the electron would spiral towards the nucleus and the atom would collapse and cease to exist within a short time, yet the atom is a stable structure. Therefore Rutherford's model can not explain the stability of the atom.

(2) Since electrons are continuously accelerating around the nucleus, continuous emission spectra should be emitted by the atom. However experimental observations reveal that it is atomic like spectra which occur.

RUTHERFORD'S α −PARTICLE SCATTERING FORMULA



Charge on α −particle$= 2e$

Charge on gold nucleus$= Ze$

b_0 is distance of closest approach

Kinetic energy of alpha particle$= \frac{1}{2}mv^2$

where v is speed before collision

Electrostatic potential energy$= \frac{Q_1Q_2}{4\pi\varepsilon_0 r} = \frac{(2e)(Ze)}{4\pi\varepsilon_0 b_O}$

At closest distance of approach $\frac{1}{2}mv^2 = \frac{(2e)(Ze)}{4\pi\varepsilon_0 b_O}$

$$\boxed{\boldsymbol{b_0} = \frac{Ze^2}{\pi\varepsilon_0 mv^2}} \quad \textbf{OR} \quad \boxed{\mathrm{K.e} = \frac{Ze^2}{2\pi\varepsilon_0 \mathbf{bo}}}$$

1.2: BOHR'S THEORY OF HYDROGEN ATOM

A bohr atom is one with a small central positive nucleus with electrons revolving around it in only certain <u>allowed</u> circular orbits and while in these orbits they do not emit radiations.

POSTULATES OR ASSUMPTIONS OF BOHR

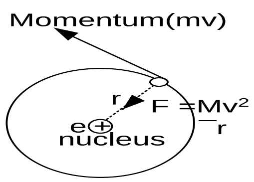
(1) Electrons revolve in <u>only allowed orbits</u> and while in these orbits they <u>do not emit radiations</u>

(2) In <u>allowed</u> orbits, the angular momentum is an electron is an integral a multiple of $\frac{h}{2\pi}$ where h is Planck's constant.$\left(mvr = \frac{nh}{2\pi}\right)$

(3) When an electron makes a transition between orbits, electromagnetic radiation of definite energy is emitted $(hf = E_4 - E_2)$

(4) In allowed orbits where the angular momentum is a multiple of $\frac{h}{2\pi}$ the energy is constant

Expression for total energy of an electron



From circular motion: Force on electron $\frac{mv^2}{r} = \frac{e^2}{4\text{л}\varepsilon_0 \mathrm{r}^2}$

$mv^2 = \frac{e^2}{4\text{л}\varepsilon_0 \mathrm{r}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots..1$

Multiplying both sides of equation (1) by mr^2

$$(mvr)^2 = \left(\frac{e^2}{4\text{л}\varepsilon_0 \mathrm{r}}\right)mr^2$$

From Bohr's assumption: $mvr = \frac{nh}{2\pi}$

$$\left(\frac{nh}{2\pi}\right)^2 = \left(\frac{mre^2}{4\text{л}\varepsilon_0}\right)$$

$r = \frac{n^2h^2\varepsilon o}{\text{л}\mathrm{m}e^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots.2$

Multiplying both sides of equation (1) by ½

$$\frac{1}{2}mv^2 = \frac{e^2}{8\text{л}\varepsilon_0 \mathrm{r}}$$

$$k.e = \frac{e^2}{8\text{л}\varepsilon_0 \mathrm{r}}$$

Also $p.e = \frac{e}{4\text{л}\varepsilon_0 \mathrm{r}} x - e$

Total energy $E = K.e + P.e$

E= $\frac{e^2}{8\text{л}\varepsilon_0 \mathrm{r}} + \frac{e}{4\text{л}\varepsilon_0 \mathrm{r}} x - e = \frac{-e^2}{8\text{л}\varepsilon_0 \mathrm{r}} \ldots\ldots\ldots\ldots\ldots.3$

Putting value of r in equation (3): $E = \frac{-e^2}{8\text{л}\varepsilon o\left(\frac{n^2h^2\varepsilon_0}{\text{л}\mathrm{m}e^2}\right)}$

$$\boxed{E = \frac{-e^4\, m}{8\mathrm{n}^2\mathrm{h}^2\varepsilon_0{}^2}}$$

Where n is quantum number
h is Planck constant
ε_0 permittivity of free space
M is mass of the electron
e is charge of electron

1.3: HYDROGEN SPECTRUM

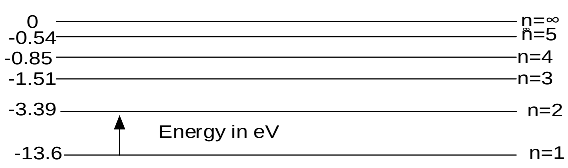
When an electron in an atom falls from one of the upper energy levels to one lower, energy is emitted in form of radiations.

The bigger the energy difference, the greater will be the energy of the emitted radiation. The frequency f of the emitted radiation is given by

$$\boxed{E = hf}$$

Where E- energy difference and h- plancks constant

A diagram of the energy levels in the hydrogen spectrum is shown below

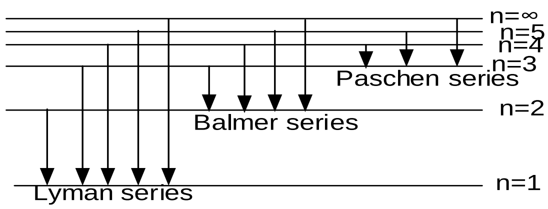


Energy $E = \frac{-13.6\ eV}{n^2}$ where $n = 1,2,3 \ldots\ldots \infty$

- ❖ The lowest level with n = 1 is called the **ground state**. The electron will always occupy this lowest level unless it absorbs energy. Ground state is also the lowest energy state for the atom.
- ❖ When the atom absorbs energy is some way, the electron may be promoted into one of the higher energy levels, the atom becomes unstable and it is said to be in **Excited state**.
- ❖ The top level with n = ∞ is the ionization state. An electron raised to this level will be removed from the atom.

HYDROGEN SPECTRAL SERIES

The spectrum of hydrogen contains distinct groups of lines known as spectral series.



- The series ending on $n = 1$ shows the largest energy transitions and gives lines in the ultra violet region of the spectrum. This is the **lyman series (ultra violet region)**.
- The series ending on $n = 2$ lies mostly in the visible region of the spectrum and is called **Balmer series (visible spectrum)**
- The series ending on $n = 3$ lies mostly in the infra red region of the spectrum and is called **Paschen series (infra red region)**

1.4: Ionisation and Excitation potential

(1) **Ionization energy** of an atom is the minimum amount of energy required to remove it's most loosely bound electron when the atom is in it's gaseous state.

(2) **Excitation energy** of an atom is the energy required to raise an electron is in it's ground state to higher energy level.

Note: If the energy absorbed is more than that for ionization then the rest appears as kinetic energy of the electrons from which it's velocity can be calculated.

Examples

1.

Calculate the frequency and wavelength of radiations resulting from the following transitions

a) $n = 4 \; to \; n = 2$ b) $n = 2 \; to \; n = 1$

In which region of the electromatic spectrum does each transition lie

Solution

(a) hf = $E_4 - E_2$

$hf = -0.85 - -3.39 = 2.54eV$

$f = \frac{2.54x \; 1.6 \; x \; 10^{-19}}{6.6x10^{-34}} = 6.16 \; x \; 10^{14}Hz$

$\lambda = \frac{C}{f} = \frac{3x10^8}{6.16x10^{14}} = 4.87 \; x \; 10^{-7}m$

Ultraviolet region

(b) hf = $E_2 - E_1$

$hf = -3.39 - -13.6 = 10.21eV$

$f = \frac{10.21x \; 1.6 \; x \; 10^{-19}}{6.6x10^{-34}} = 2.48 \; x \; 10^{15}Hz$

$\lambda = \frac{C}{f} = \frac{3x10^8}{2.48x10^{15}} = 1.21 \; x \; 10^{-7}m$

Vsible spectrum

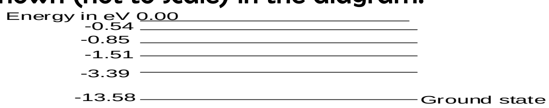
Exercise: 51

1. The ionization potential of the hydrogen atom is 13.6V. Use the data below to calculate
   (a) The speed of an electron which could just ionize the hydrogen atom.
   (b) The minimum wavelength which the hydrogen atom can emit

(charge on an electron $= 1.6\ x\ 10^{-19}$C, $h\ = 6.63x10^{-34}Js$, m $= 9.11x10^{-31}$kg, c $= 3x10^{8}\ ms^{-1}$)

An (2.19×10^{6}ms-1, 9.14×10^{-8}m)

2. Some of the energy levels of the hydrogen atom are shown (not to scale) in the diagram.

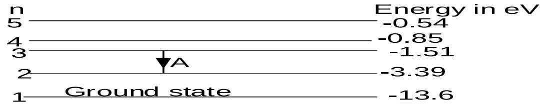


(a) Why are the energy levels labeled with negative energies

(b) State which transition will result in the emission of radiation of wavelength 487nm. Justify your answer by calculation.

(c) What is likely to happen to a beam of photons of energy (i) 12.07eV (ii) 5.25eV when passed through a vapour of atomic hydrogen

3. The diagram below represents the lowest energy levels of the electron in the hydrogen atom, giving the principal quantum number n associated with each level and the corresponding values of the energy.



(i) Why are the energies quoted with negative values

(ii) Calculate the wavelength of the line arising from the transition A, indicating in which region of the electromagnetic spectrum this occurs.

(iii) What happens when 13.6eV of energy is absorbed by a hydrogen atom in it's ground state. **An (6.6×10^{-7}m)**

4. Calculate the energy released and the wavelength of the emitted radiation when an electron falls from level $n = 3\ (-1.51eV)$ to $n = 2\ (-3.41eV)$

An (3.0×10^{-19}J, 660nm)

1.5: TYPES OF SPECTRA

Spectra are of two types;

- **Emission spectra**

  This is a spectrum in which light is given out by a source.

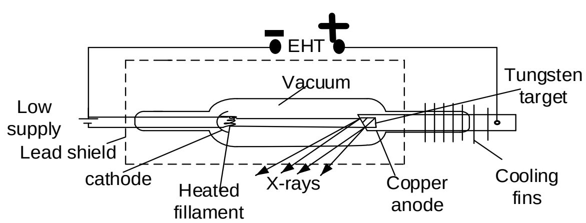
- **Absorption spectra**

  This is a spectrum in which light from a source is absorbed when it passes through another material usually a gas or a liquid.

1.6: X-RAYS

These are electromagnetic radiations of very high frequency (short wavelength) produced when cathode rays strike a metal target.

X-RAY TUBE [PRODUCITON OF X-RAY]



- The cathode is heated with low voltage and electrons are emitted thermionically.
- Electrons are accelerated by a high p.d towards the anode.
- On striking the target, a small percentage of the electron energy is converted to X-rays
- The anode is cooled by the cooling fins.

Explanation of the uses

(i) Used to detect fractures in bones

X-rays are directed to part of the body with a suspected bone fracture, the shadow of the bone is formed on a photographic film placed on the opposite side of the body

(ii) Used to destroy cancer cell

X-rays are directed to part of the body with a suspected cancer cells, the cells are then destroyed

Explanation of λ_{min}

At cut off wavelength, λ_{min} . Electrons from the cathode strike the target and lose all their kinetic energy in a single encounter with the target atoms. This results in the production of the most energetic x-ray photons of maximum frequency and corresponding, λ_{min} called cut off wavelength.

From $\boxed{E = hf}$ | $\boxed{hf_{max} = ev}$ | $\boxed{h\frac{C}{\lambda_{min}} = ev}$

Examples

1. An x-ray tube operates at 30kV and current through it is 2mA. Calculate
   (i) The electrical power input
   (ii) Number of electrons striking the target per second
   (iii) The speed of electrons when they hit the target
   (iv) The lower wavelength limit of x-rays emitted

   [h= $6.6x10^{-34}$Js, e= $1.6x10^{-19}$C, C= $3x10^{8}\ ms^{-1}$, m= $9.1x10^{-31}$kg]

   Solution

   (i) Power input $= IV = 2x10^{-3}x30x10^{3}$ =60Js^{-1}

   (ii) $I = ne$

   n= $\frac{2x10^{-3}}{1.6x10^{-19}} = 1.25x10^{16}$ electrons per second

   (iii) $u = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2x1.6x10^{-19}x30x10^{3}}{9.1x10^{-31}}} = 1.03x10^{8}ms^{-1}$

   (iv) $\lambda_{min} = \frac{h\ C}{eV} = \frac{6.6x10^{-34}x3x10^{8}}{1.6x10^{-19}x30x10^{3}} = 4.13x10^{-11}m$

2. The $p.d$ between the target and cathode of an x-ray tube is 50kV and current in the tube is 20mA. If only 1% of the total energy is emitted as x-rays.
   (i) What is the maximum frequency of the emitted radiations
   (ii) At what rate must heat be removed from the target in order to keep it a steady temperature.

Solution

i) $hf_{max} = eV$

$$f_{max} = \frac{1.6x10^{-19}x50x10^{8}}{6.6x10^{-34}} = 1.21x10^{19}Hz$$

ii) 1% of power produces x-ray, therefore 99% of power produces heat

For a steady temp the rate at which heat is supplied equals to rate at which heat is removed

Rate at which heat is supplied to the target 99% of IV

$$= \frac{99}{100}\ of\ IV = \frac{99}{100}x20x10^{-3}x50x10^{3} = 990Js^{-1}$$

1.8: X-RAY DIFFRACTION

When a parallel beam of monochromatic x-rays is incident on a crystal of interplanar separation of the same order as to the wavelength of x-rays, they are reflected from successive atomic planes, superimpose and an interference pattern is formed.

Constructive interference occurs when the path difference between x-rays scattered by successive planes is an integral multiple of the wavelength

CONDITION FOR X-RAY DIFRACTION TO OCCUR

- Wave length of x-rays must be of the same order as the interplanar spacing.
- Parallel beam of x-rays must be incident on planes

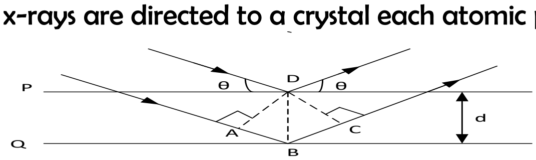
BRAGG'S LAW FOR X-RAY DIFFRACTION

Braggs law states that for constructive interference of diffracted x-rays to occur, the path difference is an integral multiple of the wavelength of x-rays. **OR**

It states that $2dsin\theta = n\lambda$ where d is interatomic spacing, θ is glancing angle, λ is x-ray wavelength and n is order of diffraction

DERIVATION OF BRAGG'S LAW FOR X-RAY DIFFRACTION

When x-rays are directed to a crystal each atomic plane of a crystal behaves like a reflecting surface.



- Constructive interference occurs when the path difference is **nλ**

Where n is an integer and λ is wavelength the x-rays.

$$\therefore AB + BC = n\lambda$$
$$AB = BC = dsin\theta$$
$$dsin\theta + dsin\theta = n\lambda$$
$$\boldsymbol{2dsin\theta = n\lambda}$$

CHAPTER 2: PHOTOELECTRIC EMISSION

It's defined as a process by which electrons are released from a clean metal surface when irradiated by electromagnetic radiations (light) of high enough frequency (energy).

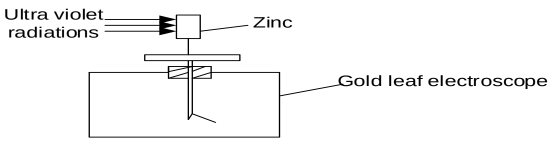
The electrons emitted this way are called **photo electrons.**

A photon: Is a packet of energy carried by electromagnetic radiations

MECHANISM OF PHOTOELECTRIC EMISSION

The radiation falling on the metal surface is absorbed by the electrons and becomes internal energy which is sufficient to enable them overcome the inward attraction for the electrons to get loose and fly off the metal surface.

2.1.0:EXPERIMENT TO DEMONSTRATE PHOTO ELECTRIC EFFECT



- A cleaned zinc plate is placed on a cap of a negatively charged gold leaf electroscope.
- When ultraviolet radiations are directed on to the plate, the leaf is seen to collapse gradually.
- This is because the plate and the cap lost charges (electrons). So the magnitude of the negative charge at the leaf and gold plate decreases thereby decreasing the divergence of the leaf gradually.

2.1.1:LAWS/RESULTS/OBSERVATIONS OF PHOTO ELECTRIC EMISSION

- For any given metal surface there is a minimum frequency of radiation called threshold frequency below which no photo electrons are emitted.
- The kinetic energies of photo electrons ranges from zero to maximum and the maximum K.E is proportional to the frequency of the incident radiation.
- The number of photo electrons emitted per second (photo current) is directly proportional to the intensity of incident radiation for a given frequency.
- There is no detectable time lag between irradiation of a metal surface and emission of electrons by the surface.

Terms used

Work function of metal (W_o): It is the minimum energy that is needed to just remove an electron from the metal surface

Threshold frequency (f_o): It is the minimum frequency of the incident radiation below which no electron emission takes place from a metal surface

Stopping potential (V_s): It is the minimum potential which reduces the photo current to zero.

Examples

1. Work function of potassium is 2.25eV. Light having wavelength of 360nm falls on the metal. Calculate;
   (i) Stopping potential
   (ii) The speed of the most energetic electron emitted
   $[h = 6.60x10^{-34}Js, c = 3x10^{8}ms^{-1}, e = 1.6x10^{-19}C]$

Solution

$W_o = 2.25eV = 2.25x1.6x10^{-19}J$, $\lambda = 360x10^{-9}m$

$$V_s = \frac{h\frac{C}{\lambda} - W_0}{e}$$

$$V_S = \frac{\frac{6.6x10^{-34}x3x10^{8}}{360x10^{-9}} - 2.25x1.6x10^{-19}}{1.6x10^{-19}} = 1.188V$$

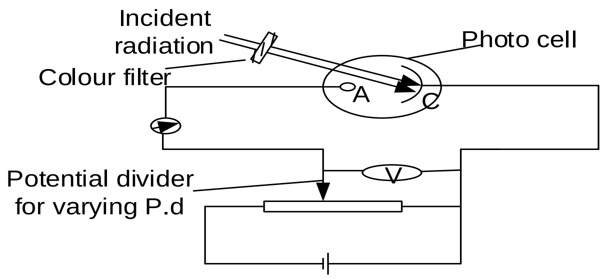
$\therefore \frac{1}{2} mv_{max}^2 = eVs$

$$V_{max} = \sqrt{\frac{2eVs}{m}} = \sqrt{\frac{2x1.6x10^{-19}x1.88}{9.1x10^{-31}}} = 6.46x10^{5}\ ms^{-1}$$

2. If a surface has a work function of 3.0eV
   (a) Find the longest wave length of light which will cause the emission of photo electrons on it.

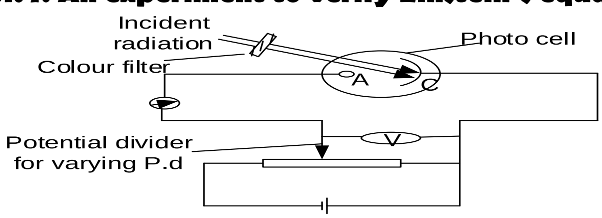
(b) What is the maximum velocity of the photo electrons liberated from the surface having a work function of 4.0eV by ultraviolet radiations of wave length 0.2μm.

2.1.3: An experiment to measure stopping potential



- The cathode C is made positive with respect to the anode by the potential divider.
- The beam of radiation is passed through a colour filter on to the cathode.
- The ammeter gives the photocurrent due to emitted electrons
- The applied p.d is increased negatively until the ammeter register zero reading.
- The $p.d$ (V_S) for which the photocurrent is zero is recorded from the voltmeter
- This p.d V_S is known as the stopping potential

2.1.4: An experiment to verify Einstein's equation or determine planck's constant



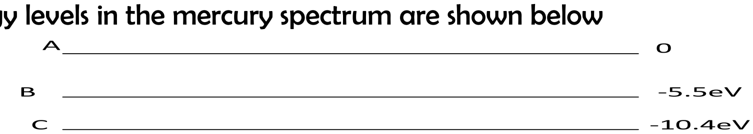
UNEB 2019 Q.9

(c) (i) An electron of charge –e and mass m moves in a circular orbit round a central hydrogen nucleus of charge +e. Derive an expression for the total energy of the electron in a n orbit of radius r. (05marks)

(ii) Why is the enrygy always negative?. (01mark)

(d) (i) What is meant by excitation potential of an atom? (01mark)

(ii) Some of the energy levels in the mercury spectrum are shown below



Calculate the wavelength of the radiation emitted when electron makes a transition from level A to level C

An$(1.19x10^{-7}m)$ (03marks)

UNEB 2018 Q.9

(a) (i) State the differences between X-rays and cathode rays. (03marks)

(ii) Describe using a labelled diagram, the mode of operation of an X-ray tube. (06marks)

(i) What is the difference between soft and hard X-rays. (01mark)

(b) (i) What is the main distinction between work function and ionisation energy (02marks)

(ii) An electron of charge, e, enters at right angles into a uniform magnetic field of flux density B and rotates at a frequency , f, in a circle of radius, r. Show that the frequency, f, is given by $f = \frac{B\,e}{2\pi m}$ (03marks)

(c) An x-ay beam is produced when electrons are accelerated through $50\ kV$ are stopped by the target of an x-ray tube. When the beam falls on a set of parallel atomic planes of a certain metal at a glancing angle of 16°, a first order diffraction maximum occurs. Calculate the atomic spacing of the planes. (05marks)

An($4.5x10^{-11}m)$

UNEB 2017 Q.9

(a) What are **X-rays?** (01mark)

(b) (i) With the aid of a diagram explain how X-rays are produced in an x-ray tube (05marks)

(ii) State the energy changes that take place in the production of X-rays in an X-ray tube (02marks)

(c) In an X-ray tube, the electron strike the target with a velocity of $3.75x10^7 m/s$ after travelling a distance of 5.0cm from the cathode. If a current of 10mA flows through the tube, find the

(i) Tube voltage **An(4003V)** (02marks)

(ii) Number of electrons striking the target per second **An(** $6.25x10^{16} electrons$) (02marks)

(iii) Number of electrons within a space of 1cm length between the anode and the cathode. (05marks) **An(** $3.3x10^7 electrons$)

(d) Briefly explain one medical application of X-rays (03marks)

UNEB 2017 Q.10

(a) State **Bohr's postulates** of the atom (03 marks)

(b) Explain the occurrence of the emission and absorption line spectra (06 marks)

(c) Explain the main observations in Rutherfordd's α −particle scattering experiment. (06 marks)

(d) A beam of alpha particles of energy 3.5 MeV is incident normal to a gold foil.

(i) Calculate the least distance of approach to the nucleus of the gold atom given its atomic number is 79. **An(** $6.5x10^{-14}m$) (04marks)

(ii) State the significance of the value of the least distance of approach (01 marks)

UNEB 2016 Q.8

(c) (i) Explain briefly diffraction of X-rays by a crystal and derive **Bragg's law**. (06marks)

(ii) A second order diffraction image is obtained by reflection of X-rays at atomic planes of a crystal for a glancing angle of $11°24'$. Calculate the atomic spacing of the planes if the the wavelenght of X-rays is $4.0x10^{-11}m$. **An (**$2.02x10^{-10}m$) (06marks)

UNEB 2016 Q.9

(a) State **Bohr's model** of an atom (02 marks)

(b) An electron of mass, m and charge, -e, is considered to move in circular orbit about a proton

(i) Write down the expression for the force on the electron. (02marks)

(ii) Derive an expression for the total energy of the electron given the angular momentum of the electron is equal to $\frac{nh}{2\pi}$ where n is an integer and h is plancks constant. (06 marks)

(c) With the aid of a labelled diagram, describe the operation of the diffusion cloud chamber. (06 marks)

(d) The energy levels of an atom have values

E_1 = - 21.4 eV
E_2 = - 4.87 eV
E_3 = - 2.77 eV
E_4 = - 0.81 eV
E_∞ = - 0.00 eV

(iii) Calculate the wavelength of the radiation emitted when an electron makes a transition from E_3 and E_2 **An(**$5.89x10^{-7}m$**.)** (03 marks)

(iv) State the region of the electromagnetic spectrum where the radiation lies. (01 marks)

UNEB 2015 Q.9

(a) (i) State the laws of photoelectric emission (04 marks)

(ii) Explain briefly one application of photoelectric effect (04marks)

(b) In a photoelectric set up, a point source of light of power $3.2x10^{-3}W$ emits mono-energetic photons of energy 5.0eV. The source is located at a distance of 8.0m from the center of a stationary metallic sphere of work function 3.0eV and of radius $8.0x10^{-3}m$. The efficiency of photoelectron emission is one in every 10^6incident photons. Calculate the ,

(i) Number of photoelectrons emitted per second (04 marks)

(ii) Maximum kinetic energy in joules, of the photo electrons (02 marks)

(c) (i) State Braggs law of X-ray diffraction (01 marks)

(ii) Show that density ρ, of a crystal can be given by $\rho = \frac{Msin^3\theta}{125\ N_A(n\lambda)^3}$

where θ is the glancing angle, n is the order of diffraction, $\lambda$is the x-ray wavelength and M is molecular weight of the crystal (05 marks)

Solution

(b) (i) Number of photons emitted per second by the lamp= $\frac{3.2x10^{-3}}{5x1.6x10^{-19}} = 4.0x10^{15}$

Number of photons incident per second on the sphere= $\frac{4.0x10^{15}x\ \pi x\ (8.0x10^{-3})^2}{4\pi x(0.8)^2} = 1.0x10^{11}$

Number of electrons emitted per second = $\frac{1.0x10^{11}}{10^6} = 1.0x10^5$

(iii) Max k.e $= 5 - 3 = 2eV$

$= 2x1.6x10^{-19} = 3.2x10^{-19}J$

UNEB 2014 Q.8

(a) State **Rutherford's model** of the atom (02marks)

(b) Explain how Bohr's model of the atom addresses the two main failures of Rutherford's model

UNEB 2014 Q.9

(a) What is **photo electric emission** (01marks)

(b) **(i)** Describe a simple experiment to demonstrate photo electric effect (04marks)

(iv) When a clean surface of metal in a vacuum is irradiated with light of wavelength $5.5x10^{-7}m$, electrons just emerge from the surface. However when light of wavelength $5.0x10^{-7}m$ is incident on the metal surface, elections are emitted each with energy$3.62x10^{-20}J$. Find the value 0f Planck's constant **An**($6.64x10^{-34}Js^{-1}$) (04marks)

(c) **(i)** With the aid of a labelled diagram, describe an X-ray tube and how X-rays are produced (05marks)

(ii) Describe how the intensity and quality of X-rays is controlled in an X-ray tube. (02marks)

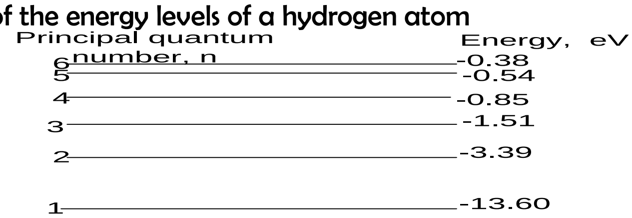
(d) An X-ray tube operated at $\mathbf{1.5x10^{-3}V}$ and the current through it is $\mathbf{1.0x10^{-3}A}$. Find the,

(i) Number of electrons crossing the tube per second. **An**($6.25x10^{15}s^{-1}$) (02marks)

(ii) Kinetic energy gained by electrons travelling the tube. **An**($2.4x10^{-22}J$) (02marks)

UNEB 2013 Q.9

(a) Figure shows some of the energy levels of a hydrogen atom



(i) Why are the energies for the different levels negative (01marks)

(ii) Calculate the wavelength of the line arising from a transition from the third to the second energy level **An (6.6×10^{-7}m)** (03marks)

(iii) Calculate the ionization energy in joules of hydrogen **An ($2.176x10^{-18}J$)** (02marks)

(b) Explain the physical process in an X-ray tube that accounts for

(i) Cut off wavelength (03marks)

(ii) Characteristic line (04marks)
(c) Calculate the maximum frequency of radiation emitted by an X-ray tube using an accelerating voltage of 33.0kV **An ($8x10^{18}Hz$)** (03marks)
(d) Derive Bragg's law of X-ray diffraction in crystal (04marks)

UNEB 2013 Q.10

(a) A beam of $\alpha - particles$ directed normally to a thin metal foil. Explain why
(i) Most of the $\alpha - particles$ passed straight through the foil (02marks)
(ii) Few $\alpha - particles$ are deflected through angles more than $90°$ (02marks)
(b) Calculate the least distance of approach of **a** $3.5MeV$ $\alpha - particles$ to the nucleus of a gold atom (atomic number of gold = 79) **An($6.495x10^{-14}m$)** (04marks)

UNEB 2011 Q.9

a) (i) Explain how X-rays are produced in an X-ray tube
(ii) Explain the emission of X-ray characteristic spectra (03marks)
(iii) Derive the Bragg X-ray diffraction equation (04marks)
(iv) Under what conditions does X-ray diffraction occur (02marks)

UNEB 2010 Q.10

(e) (i) show that when an alpha particle collides head on with an atom of atomic number. The closest distance of approach to the nucleus, Z_0 is given by $Z_0 = \frac{Ze^2}{\pi\varepsilon_0 mv^2}$

Where e is the electronic charge ε_0 is the permittivity of free space, m is the mass of the alpha particle and V is the initial speed of the alpha particle (04marks)

UNEB 2009 Q.10

(a) (i) Explain the observations made in the Rutherford's alpha particle scattering experiment (06marks)
(ii) Why is a vacuum necessary in this experiment (01mark)
(b) Distinguish between excitation and ionization energies of an atom (02marks)
(c) Draw a labeled diagram showing the main components of an X- ray tube. (03marks)
(d) An X-ray tube is operated at 50kV and 20mA. If 1% of the total energy supplied is emitted as X-radiation, calculate the;
(i) Maximum frequency of the emitted radiation (3mk)
(ii) Rate at which heat must be removed from the target in order to keep it at a steady temperature (03marks)
(e) A beam of X-rays of wavelength 0.20nm is incident on a crystal at a glancing angle of 30°. If the inter planar separation is 0.20nm, find the order of diffraction.

(An (1.21×10^{19}Hz, 990W, n = 1 (first order diffraction))

UNEB 2008 Q.8

(a) What is meant by a line spectrum (02marks)
(b) Explain how line spectra accounts for the existence of discrete energy level in atoms (4mk)
(d) Describe with aid of a labeled diagram, the action of an X-ray tube
(e) An X-ray tube is operated at 20kV with an electron current of 16mA in the tube estimate the;
(i) Number of electrons hitting the target per second (02marks)
(ii) Rate of production of heat, assuming 99.5% of the kinetic energy of electrons is converted to heat (e = 1.6×10^{-19}C) **An (1.0×10^{17} electron per second, 318.4W)** 02mks)

CHAPTER 3: NUCLEAR STRUCTURE

The nucleus is the central positively charged part of an atom.
Nuclei contain protons and neutrons which are collectively referred to as **nucleons** (**nuclear number**).

3.1.0: ATOMIC NUMBER Z, MASS NUMBER A AND ISOTOPES

Atomic number Z of an element is the number of protons in the nucleus of an atom of the element.

Mass number A of an atom is the number of nucleons in its nucleus.

Isotopes are atoms of the same element with the same atomic number but different mass numbers.
Isotopes of an element whose chemical symbol is represented by X can be distinguished by using the symbol $^{A}_{Z}X$

3.1.1: EINSTEIN'S MASS – ENERGY RELATION

Einstein showed from his theory of relativity that mass (m) and energy (E) can be changed from one form to another.
The energy Δ**E** produced by a change of mass Δ**M** is given by the relation.

$$\boxed{\Delta \mathrm{E} = \Delta \boldsymbol{MC^2}}$$

Where C is the speed of light ($C = 3\ x\ 10^{8} ms^{-1}$)

3.1.2: UNIFIED ATOMIC MASS UNIT [u]

It is defined as $^{1}/_{12}$ of the mass of carbon–12 atom.

mass$= \frac{number\ of\ atoms}{N_A} x\ molar\ mass$

mass $= \frac{1x12\ x\ 10^{-3}}{6.02x10^{23}} = 1.993x10^{-26}$

1unified atomic mass $= {}^{1}/_{12}\ x\ 1.993x10^{-26}$

$$\boxed{1u = 1.66\ x\ 10^{-27}kg}$$

From Einstein's mass – energy relation: $\Delta E = MC^2$

$1u \quad = 1.66x\ 10^{-27}\ x\ (3\ x\ 10^{8})^{2}$

$1u = 1.494\ x\ 10^{-10}J$

$1eV = 1.6x10^{-19}J$

$1u = \frac{1.494\ x\ 10^{-10}}{1.6x10^{-19}} eV$

$= 933.75\ x\ 10^{6}eV$

$$\boxed{1u = 931MeV}$$

3.1.3: MASS DEFECT AND BINDING ENERGY

a) MASS DEFECT

It is defined as the mass equivalence of the energy required to split the nucleus into its constituent particles.
OR
It is the difference in the mass of the constituent nucleons and the nucleus of an atom.

$$\boxed{\boldsymbol{Mass\ defect = (mass\ of\ nucleons) - (mass\ of\ atom)}}$$

b) BINDING ENERGY (B.E)

- Binding energy of the **nucleus** is the energy required to break up the nucleus into it's constituent nucleons
- Bind energy per nucleon is the ratio of the energy needed to split a nucleus into it's constituent nucleons to the mass number.

$$\boxed{\mathbf{B.E\ per\ nucleon} = \frac{\boldsymbol{B\ E}}{\mathbf{Mass\ number}}}$$

Examples

1. Given atomic mass of $^{238}_{92}\boldsymbol{U} = 238.05076u$
   $mass\ of\ neutron = 1.00867u,\ mass\ of\ proton = 1.00728u$ and $1u = 931MeV$

Find; a) mass defect b) B.E per nucleon for $^{238}_{92}U$

Solution

Δm =(mass nucleons)−(mass of nucleus)

number of protons $= 92$

number of neutrons $= (238 - 92) = 146$

$$\Delta m = (146 \, x \, 1.00867 + 92 \, x \, 1.00728) - (238.05076)$$

$$= 239.93558 - 238.05076$$

$$\Delta m = 1.88482u$$

b) B.E per nucleon $= \frac{B\,E}{\text{Mass number}}$

B.E $= mass\ defect\ x\ 931MeV$

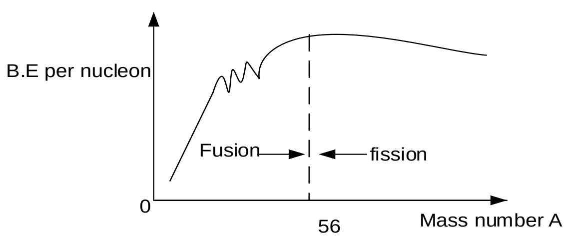
$= 1.88482\ x\ 931 = 1754.77MeV$

B.E per nucleon $= \frac{1754.77}{238} = 7.373MeV$

EXERCISE 55

1. Given the mass of the nucleus of the isotope $^{7}_{3}\boldsymbol{Li} = 7.014351u$
   $mass\ of\ neutron = 1.008665u$
   $mass\ of\ proton = 1.007275u$
   $1u = 931MeV$
   Find the binding energy per nucleon.
   An(5.586MeV)
2. Given the mass of the nucleus of the isotope $^{16}_{8}\boldsymbol{O} = 15.994915u$
   $mass\ of\ neutron = 1.008665u$
   $mass\ of\ proton = 1.007275u$
   $1u = 931MeV$
   Find the binding energy per nucleon.
   An(7.97MeV)

3.1.4: VARIATION OF B.E PER NUCLEON WITH MASS NUMBER



- Binding energy per nucleon for very small and large nuclides is small.
- A few peaks for low mass numbers are for lighter nuclei that are comparatively stable.
- The binding energy per nucleon increases sharply to a maximum at mass number 56
- For A> 56 binding energy per nucleon gradually decreases

3.1.5: Explanation of fusion and fission using the graph

- **During nuclear fusion** two light nuclei unite to form a heavier nucleus of a smaller mass but a higher binding energy per nucleon. The mass difference is accounted for by the energy released.
- **During Nuclear fission**, a heavy nucleus splits to form two lighter nuclei of smaller masses but a higher binding energy per nucleon. The mass difference is accounted for by the energy released

3.2.0: RADIO-ACTIVITY (RADIOACTIVE DECAY)

Radioactivity is the random and spontaneous disintegration of a radioactive atom into more stable nuclei with emission of radiations.

Note:

(i) Heavy nuclides are generally unstable if there are too many neutrons or too many protons. This is because too many protons increases electrostatic repulsion between themselves. This force may not be counter balanced by the nuclear force. Hence nucleus becomes unstable

(ii) Radioactive decay is random because it is impossible to predict which particular nucleus will decay next and radioactive decay is spontaneous because you cannot influence the rate of decay by physical and chemical change.

RADIOACTIVE –ISOTOPES

Are radioactive atoms of the same element with the same atomic number but different mass numbers

USES OF RADIOACTIVITY(radio-active isotopes)

- Treatment of cancer
- Used in carbon dating
- Detection of leaks in pipes
- Production of energy in nuclear reactors
- Measurement of thickness of metal sheet during manufacture
- In automobile industry to test the quality of steel in manufacture of cars
- Tracers to investigate flow of fluids in chemical plants
- In construction to gauge the density of the road surface

3.2.1: ENERGY OF DISINTEGRATION (Q-value)

If the total mass of reactant is greater than the total mass of products then the reaction is **exothermic** otherwise its **endothermic**

Examples

2. Consider the equation $^{206}_{82}Pb + Q \rightarrow \ ^{4}_{2}He \quad + ^{202}_{80}Hg$

Atomic mass of $Hg \ = \ 201.971u$

Atomic mass of $He = \ 4.003u$

Atomic mass of P$b \ = \ 205.969u$

Calculate i) Q –value ii) kinetic energy of the α-particle

Solution

i) $Q \ = \ mass \ x \ 931MeV$

$Q \ = \ ((201.971 \ + \ 4.003) - 205.969) \ x \ 931MeV$

Q-value = $4.66MeV$

ii) $K.e_{\alpha} = \frac{M}{M+m_{\alpha}}Q$

$K.e_{\alpha} = \left(\frac{202}{202+4}\right)4.66 \ = \ 4.57MeV$

EXERCISE 55

1. $^{210}_{84}Po$ decays to $^{206}_{82}Pb$ by emission of α – particles of single energy
   (i) Write down the symbolic equation for the reaction
   (ii) Calculate the energy in MeV released in each disintegration
   (iii) Explain why this energy does not all appear as kinetic energy of the alpha particle.
   (iv) Calculate the kinetic energy of the alpha particle

   $210Po = \ 209.93673u$

   $206Pb = 205.929421u$

   $4He = 4.001504u$

   $1u = 931MeV$ **An (5.40MeV, 5.30MeV)**

2. Beta particle emmission from $^{210}_{83}Bi$ can be described by the equation

   $^{210}_{83}Bi \ \rightarrow \ ^{210}_{84}Po + \ ^{0}_{-1}e + \gamma + Q$

   Mass of $^{210}_{83}Bi = 209.98411u$,

   of $^{210}_{84}Po \ = \ 209.982866u$

   Calculate the value of Q
   (i) In Joules
   (ii) In MeV.

   An ($1.9x10^{-13}J, 1.19MeV$)

3. A nucleus $^{23}_{10}Ne, \beta - decays$ to give the nucleus of $^{23}_{10}Na$.

   Mass of $^{23}_{10}Ne = 22.994466u$,

   of $^{23}_{11}Na \ = \ 22.989770u$

   Calculate the energy of the emitted electron.

   An($4.37MeV$)

3.2.3: NUCLEAR FISSION

Nuclear fission is the disintegration of a heavy nucleus into two lighter nuclei accompanied by release of energy.

Application of fission

- In the production of neutrons
- In production of atomic bombs

Condition for fission

- It requires an energetic particle like a neutron

Example

Calculate the energy released by 1kg of $^{235}_{92}U$ under going fission according to

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{148}_{57}La + ^{85}_{35}Br + 3\,^{1}_{0}n + Q$$

Mass of $235U = 235.1u$, Mass of $148La = 148.0u$, Mass of $^{1}_{0}n = 1.009u$, Mass of $85Br = 84.9u$

Solution

Mass of reactants $= 235.1 + 1.009 = 236.109u$

Mass of products $= (148.0 + 84.9 + (3 \times 1.009))$

$= 235.927u$

Energy released $= mass\ defect \times 931MeV$

$= (236.109 - 235.927) \times 931MeV$

$= 169.442MeV$

Energy released $= 169.442 \times 10^{6} \times 1.6 \times 10^{-19}$J

$= 2.71 \times 10^{-11}$J

Number of atoms $= \frac{m}{M}N_A$ atoms

1 kg contains $= \frac{1x6.02 \times 10^{23}}{235 \times 10^{-3}} = 2.562 \times 10^{24}$ atoms

One atom released $= 2.71 \times 10^{-11}$J

2.562×10^{24} atoms $= 2.71 \times 10^{11} \times 2.562 \times 10^{24}$J

$= 6.943 \times 10^{13}$J

Energy released by 1kg of uranium $= 6.943 \times 10^{13}$J

3.2.4: NUCLEAR FUSION

Nuclear fusion is the union of two light nuclei to form a heavier nucleus accompanied by release of energy. Energy is released in the process.

Condition for fusion

- High temperatures (in excess of 10^{8}K) are required to provide the nuclei which are to fuse with the energy needed to overcome their mutual electrostatic repulsion.

Note

- Fusion is the basis of hydrogen bond
- Solar energy is produced by the process of fusion.

Example

1. $^{2}_{1}H + ^{2}_{1}H \rightarrow ^{3}_{2}He + ^{1}_{0}n$

Calculate the amount of energy released by 2kg of Deuterium given

$$(2H = 2.015u, 1n = 1.009u, 3He = 3.017u)$$

Solution

Mass of reactant $= 2.015 + 2.015 = 4.03u$

Mass of products $= 3.017 + 1.009 = 4.026u$

Mass defect $= 4.03 - 4.026 = 0.004u$

Energy released $= Mc^2 = 0.004 \times 1.66 \times 10^{-27} \times (3 \times 10^{8})^{2}$

$= 5.976 \times 10^{-13}$J

Energy released by 2 atoms of $^{2}_{1}H = 5.976 \times 10^{-13}$J

Energy released by 1 atom of $^{2}_{1}H = \frac{5.976 \times 10^{-13}}{2}$

Energy released by 1 atom $^{2}_{1}H = 2.988 \times 10^{-13}$J

Number of atoms $= \frac{m}{M}N_A$ atoms

$= \frac{2x6.02 \times 10^{23}}{2 \times 10^{-3}} = 6.02 \times 10^{26}$ atoms

1 atom of $^{2}_{1}H = 2.988 \times 10^{-13}$J

6.02×10^{26} atoms $= 2.988 \times 10^{-13} \times 6.02 \times 10^{26}$

$= 1.799 \times 10^{14}$J

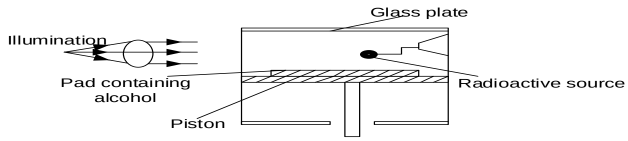
Energy released by 2kg $= 1.799 \times 10^{14}$J

3.2.6: DETECTION OF IONISING RADIATIONS

1. CLOUD CHAMBERS

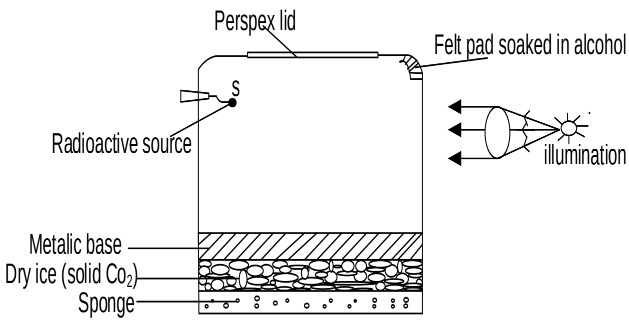
The cloud chamber is used to show tracks of the radioactive particles rather than to measure the intensity of the cloud chambers are;

(a) Wilson cloud chamber



- The piston is moved down quickly so that the air in the chamber undergoes an adiabatic expansion and cools.
- Dusts particles are carried away by drops forming on air after a few expansions. The dust free air is subjected to a controlled adiabatic expansion, where by it becomes super saturated and it is exposed to the radioactive source.
- Water droplets collect round the ions producing tracks viewed through the glass plate

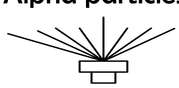
(b) DIFFUSION CLOUD CHAMBER



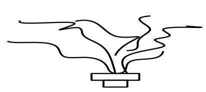
- The base of the chamber is maintained at a very low temperature by solid carbondioxide and air in the upper part of the chamber is at room temperature so that there is a temperature gradient between the top and bottom.
- The air in the chamber is saturated with alcohol vapour from the felt pad. The vapour diffuses downwards into the cooler region until the air above becomes supersaturated with alcohol.
- The radioactive source, S is opened and ionises the air molecules
- The saturated vapour condenses on the ion formed. The path of the ionizing radiations is traced by a series of drops of condensation.
- The thickness and length of the path indicates the extent to which ionization has taken place.

Nature of the tracks in cloud chamber

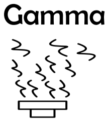
(i) Alpha particles produce thick, short and straight continuous tracks



(ii) Beta particles produce, thin longer and wavy tracks

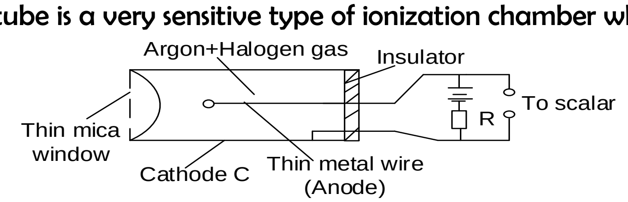


(iii) Gamma rays produce irregular and faint tracks



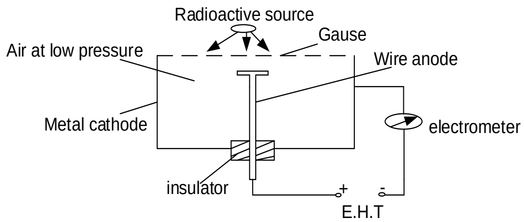
2. THE GEIGER – MULLER TUBE / (GM) TUBE

Gm tube is a very sensitive type of ionization chamber which can detect single ionizing events



- When Ionising radiations enter the G.M tube through the thin mica window, argon atoms are ionised
- The electrons move very fast to the anode and the positive ions drift to the cathode.
- When electrons reach anode, a discharge occurs and a current flows in the external circuit.
- A p.d is obtained across a large resistance R which is amplified and passed to a scale
- The magnitude of the pulse registered gives the extent to which ionisation occurred.

3. THE IONISATION CHAMBER



- Ionizing radiations enter through the thin wire gauze and ionises the air molecules.
- The ions produced are accelerated by E.H.T to their respective electrodes
- The electrons move towards the anode and the positive ions towards the cathode.
- Current flows in the external circuit which is amplified and detected by electrometer
- The pulse per second (count rate) gives a measure of the intensity of radiation

3.2.7: THE RADIOACTIVE –DECAY LAW [$N = N_0 e^{-\lambda t}$]

Activity is the number of decays per second. OR it is the number of radiations emitted per second.

$$\boxed{A = \lambda N}$$

Where A is activity or count rate per second. The S.I unit for activity (A) is Becquel (Bq)

Decay constant is the fraction of radioactive atoms which decay per second.

3.2.8: HALF LIFE [$t_{\frac{1}{2}}$]

Half life of a radioactive element is the time taken for half of the atoms to decay

Relation between half life and decay constant

If N_0 is the number of original atoms

at $t = t_{\frac{1}{2}}, N = \frac{N_0}{2}$

From $N = N_0 e^{-\lambda t}$

$$\frac{N_o}{2} = N_o e^{-\lambda t_{1/2}}$$

$$\frac{1}{2} = e^{-\lambda t_{1/2}}$$

Taking logs to base e on both sides

$$In(\frac{1}{2}) = Ine^{-\lambda t_{1/2}}$$

$$In\left(\frac{1}{2}\right) = -\lambda t_{1/2}$$

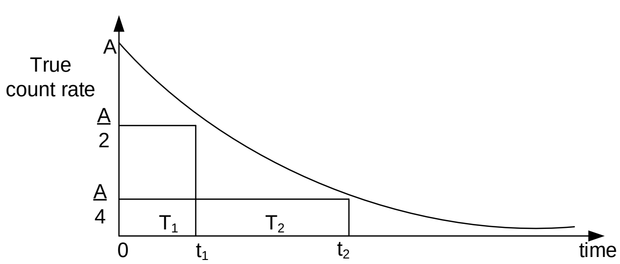
$$t_{1/2} = \frac{-In(1/2)}{\lambda} = \frac{In2}{\lambda}$$

$$\boxed{t_{1/2} = \frac{0.693}{\lambda}}$$

Note: Activity A at any one given time t is given **by** $A = A_0 e^{-\lambda t}$

Measurement of half-life

(a) Half-life of short lived isotopes



- Switch on the G.M.T, note and record the background count rate A_O .
- Place a source of ionising radiation near the GM-tube window. Note and record the count rate at equal time intervals
- For each count rate recorded, subtract the background countrate to get true count rate.
- A graph of true count rate against time is plotted
- Find the time T_1 taken for acitivity to reduce to $\frac{A}{2}$ and time T_2 taken for activity to reduce to $\frac{A}{4}$ from $\frac{A}{2}$. Half life=$\frac{1}{2}(T_1 + T_2)$

Note: Background count rate is the activity detected by GM-tube in the absence of a radioactive source

Examples

1. An isotope of krypton $^{87}_{36}Kr$ has a half-life of 78 minutes. Calculate the activity of 10μg of $^{87}_{36}kr$

Solution

Number of atoms $= \frac{m}{M}N_A$ atoms

$= \frac{6x10^{23}}{87}x10x10^{-6}$ =6.9x10^{16}atoms

But $A = \lambda N = \frac{In2}{78x60}x6.9x10^{16}$

A = 1.022x10^{13}Bq

2. What mass of radium -227 would have an activity of 1x10^{6}Bq. The half life of radium-227 is 41minutes (N_A=6x10^{23}mol^{-1})

Solution

$A = \lambda N$

1x10^6 $= \left(\frac{In2}{41x60}\right)N$

N = 3.55x10^{9}atoms

Number of atoms $= \frac{m}{M}N_A$ atoms

$m = \frac{227}{6x10^{23}}x3.55x10^9 = 1.34x10^{-12}g$

3. A radioactive source contains 1.0μg of plutonium of mass number 239. If the source emits 2300 alpha particles per second. Calculate the half life of plutonium, assume [$N = N_0e^{-\lambda t}$]

Solution

Number of atoms $= \frac{m}{M}N_A$ atoms

$= \frac{6.02x10^{23}}{2.39}x10^{-6}$ = 2.519 x 10^{15}atoms

Since it emits 2300 alpha particles per second, then

$A = 2300s^{-1}$

$A = \lambda N$

2300 = $\lambda x2.519x10^{15}$

2300 = $\left(\frac{In2}{t_{1/2}}\right)x2.519x10^{15}$

$t_{1/2} = \frac{2.519x10^{15}In2}{2300} = 7.591x10^{11}s$

4. A small volume of a solution which contains a radioactive isotope of sodium had an activity of 12000 disintegration per minute when it was injected into a blood stream of a patient. After 30 hours, the activity of 1.0cm^3 of the blood was found to be 0.50 disintegration per minute. If the half life of the sodium isotope is taken as 15 hours, estimate the volume of blood in a patient

Solution

$A = A_Oe^{-\lambda t}$

$A = 12000e^{-\frac{In2}{15}x30} = 3000min^{-1}$

Total volume of blood = $\frac{Activity\ in\ the\ blood\ stream}{activity\ in\ 1cm^3}$

$= \frac{3000}{0.5}$

$= 6000cm^3$

Therefore volume of blood in a patient= 6litres

3.2.9: CARBON DATING

- Carbon-14 is radioactive with half life , $t_{1/2} = 5600\ years$. It is absorbed by plants during photosynthesis, when plants dies carbon-14 starts to decay.
- The activity, A_O of living plants is measured. The acitivity A of dead plants is also measured. The age, t of the dead plant is deduced from $A = A_Oe^{-\lambda t}$ where $\lambda = \frac{In2}{t_{1/2}}$

Examples

1. Wood from a buried ship was forced to have a specific activity of $1.2x10^2\ BqKg^{-1}$due to ^{14}C whereas a comparable living wood has a specific activity of $2x10^2\ BqKg^{-1}$. What is the age of the ship? [half life of $^{14}C = 5.7x10^3years$]

Solution

$A = A_Oe^{-\lambda t}$

$1.2x10^2 = 2x10^2e^{-\frac{In2}{t_{1/2}}t}$

$In(\frac{1.2}{2}) = t\frac{-In2}{(5.7x10^3)}$

$t = 4.2x10^3years$

2. A radioactive source has a half life of 20s and an initial activity of $7x10^{12}\ Bq$. Calculate its activity after 50s have elapsed

Solution

$\boldsymbol{A = A_0e^{-\lambda t}}$

$A = 7x10^{12}\ e^{\frac{-In2}{20}x50} = 1.24x10^{12}Bq$

EXERCISE: 56

1. A certain $\alpha - particle$ the track in a cloud chamber has length of 37mm. Given that the average energy required to produce an ion pair in air is 5.2x10^{-18}J and that $\alpha - particles$ in air produce on average 5x10^{3} such airs per mm of track. Find the initial energy of the $\alpha - particle$. Express your answer in MeV [e=1.6x10^{-19}C] **An(6.0MeV)**
2. Calculate the count rate produced by $0.1\mu g$ of caesium-137(The half of Cs-137=28years) **An($3.45x10^5$Bq)**
3. The radioactive isotope $^{218}_{84}P_o$ has a half life of 3minutes, emitting $\alpha - particles$ according to the equation;

$$^{218}_{84}Po \rightarrow \alpha + \,^{x}_{y}Pb$$

   (i) What are the values of x and y
   (ii) If N atoms of $^{218}_{84}P_o$ emit $\alpha - particles$ at a rate of 5.12x10^{-4}s^{-1}, what will be the rate of emission after $^1/_2$ hour. **An(50s^{-1})**
4. An isotope of the element radon has a half life of 4days . A sample of radon originally contains 10^{10} atoms.[Take 1day to be 86x10^{3}s]. Calculate;
   (i) The number of radon atoms remaining after 16days
   (ii) The radioactive decay constant for radon
   (iii) The rate of decay of the radon sample after 16days
   An(6.3x10^{8}atoms, 2x10^{-6}s^{-1}, 1.3x10^{3}Bq)
5. (a) What is meant by the decay constant λ and the half life $T_{1/2}$ for a radioactive isotope?.
   Show from first principles that $\lambda T_{1/2}$= 0.69
   (b) At a certain time, two radioactive sources R and S contain the same number of radioactive nuclei. The half life is 2hours for R and 1 hour for S, calculate
   (i) The ratio of the rate of decay of R to that of S at this time
   (ii) The ratio of the rate of decay of R to that of S after 2 hours
   (iii) The proportion of the radioactive nuclei in S which have decayed in 2 hours **An [1:2, 1:1, 75%]**
6. The isotope of bismuth of mass number 200 has a half life of 5.4x10^{3}s. It emits alpha particles with an energy of 8.2x10^{-13}J.
   (a) State the meaning of the term half life
   (b) Calculate for this isotope;
   (i) Decay constant
   (ii) The initial activity of 1x10^{-6} mole of the isotope
   (iii) the initial power output of this quantity of the isotope
   [N_A = 6x10^{23}mol^{-1}] **[Hint, power = activity x Energy] [An 1.3x10^{-4}s^{-1}, 7.7x10^{13}s^{-1}, 63W]**
7. The radioactive isotope ^{60}Co decays to ^{60}Ni which spontaneously decays to give two gamma-ray photons, the half life of ^{60}Co is 5.27years.
   (i) find the activity of 20g of ^{60}Co
   (ii) estimate the power obtainable from 20g of ^{60}Co
   [Mass of ^{60}Co= 59.93381U, mass of ^{60}Ni = 59.93079U] **[An 8.35x10^{14}s^{-1}, 3.76x10^{2}Js^{-1}]**

UNEB 2020 Q.8

(a) What is meant by the following as applied to radioactivity? .
   (i) Activity. (01mark)
   (ii) Decay constant (01mark)
(b) (i) Explain briefly, why radioactivity is refered to as random and spontaneous. (02marks)
   (ii) The half life of $^{230}_{92}Th$ is $2.4x10^{11}s$. Find the number of disintegrations persecond that occur in 1g of $^{230}_{92}Th$ **An(7.559x10^{9}s^{-1})** (05marks)
(c) (i) Describe, with the aid of a labelled diagram, how the Wilson cloud chamber can be used to detect ionizing radiation. (06marks)
   (iii) Explain the difference in the patterns of the tracks seen in the chamber when $\alpha -$ and $\beta -$particles are present in the chamber. (02marks)
(d) (i) What is meant by **mass defect** (01mark)
   (ii) Calculate, in MeV , the energy released when a helium nucleons are produced by fusing two protons and two neutrons. (04marks)

Mass of a $neutron = \ 1.00898u$
Mass of $a\ proton = \ 1.00759u$

Mass of $helium = 4.00277u$
$(1u = 931MeV)$ **An(28.2745MeV)**

UNEB 2019 Q.10

(a) What is meant by the following as applied to radioactivity? .
(i) Activity. (01mark)
(ii) Half life of a radioactive material. (01mark)

(b) Using the radioactive decay law $N = N_o e^{-\lambda t}$, show that the half-life $T_{1/2}$ is given by:

$$T_{1/2} = \frac{0.693}{\lambda}$$ (03marks)

(c) With the aid of a diagram, describe action of an ionization chamber. (06marks)

(d) What is meant by unifed **atomic mass unit** and **electron volt** (02marks)

(e) (i) The nucleus $^{212}_{83}Bi$decays by alpha emission as follows $^{212}_{83}Bi \rightarrow ^{208}_{81}Ti + ^{4}_{2}He$
Calculate the energy released by 2g of $^{212}_{83}Bi$ (05marks)
(ii) Explain two uses of radiactive isotopes. (04marks)

UNEB 2018Q.8

(a) Define the following .
(i) Binding energy. (01mark)
(ii) Unified Atomic Mass Unit. (01mark)

(b) Explain how energy is released in a nuclear fusion process. (03marks)

UNEB 2018Q.10

(a) State **two** differences between **alpha** and **beta** particles. (02marks)

(b) Describe with the aid of a diagram, the structure and mode of the operation of an ionization chamber. (06marks)

(c) (i) Explain the application of carbon-14 in carbon dating. (03marks)
(ii) A sample of dead wood was found to have an activity of 20 units due to ^{14}C. Recent wood gave an activity of 47.8units, estimate the age of the wood [half life of ^{14}C = 5600years]. **An**. $7.04x10^3 years$ (03marks)

(d) The photoelectric work function of potassium is $2.25eV$. Light having a wavelength of 360mm falls on potassium metal.
(i) Calculate the stopping potential (04marks)
(ii) Calculate the speed of the most energetic electrons emitted by the metal. (02marks)
Ans $[-2.25V, 8.89x10^5 m/s]$

UNEB 2017Q.8

(a) What is meant by the following.
(i) Radioactivity. (01mark)
(ii) Isotopes (01mark)

(b) (i) Define **mass defect**. (01mark)
(ii) State the condition for a heavy nucleus of an atom to be unstable. (01mark)
(iii) Explain your answer in (b) (ii) (02marks)

(c) A sample of $^{226}_{88}Ra$ emits both α −particles and $\gamma - rays$. A masss defect of 0.0053u occurs in the decay
(i) Calculate the energy released in joules **Ans** $[7.92x10^{-13}J]$ (03marks)
(ii) If the sample decays by emission of α −particles, each of energy 4.60MeV and $\gamma - rays$, find the frequency of the $\gamma - rays$ emitted. **Ans** $[8.5x10^{19}Hz]$ (04marks)

(d) (i) Sketch a graph showing the variation of binding energy per nucleon with mass number, clearly showing the fusion and fission regions (02marks)
(ii) Use the sketch in (d) (i) to explain how energy is released in each of the processes of fusion and fission (03marks)

(e) State **two**
(i) Applications of radioisotopes (01mark)

(ii) Health hazards of radioisotope (01mark)

UNEB 2016 Q.8

(b) (i) Distinguish between **mass defect** and **binding energy**. (02mark)

(ii) Sketch a graph of nuclear binding energy per nucleon versus mass number of naturally occurring isotopes and use it to distinguish between nuclear fission and fussion. (04marks)

(c) Describe with the aid of labelled diagram, Milikan's oil drop experiment to determine charge on an oil drop. (07marks)

UNEB 2015 Q.10

(a) with reference to a Geiger-Muller tube, define the following

(i) quenching agent (01mark)

(ii) back ground count rate (01mark)

(b) (i) with the aid of a labelled diagram, describe the operation of Geiger-Muller tube (01mark)

(ii) Explain how the half-life of a short lived radioactive source can be obtained by use of a Geiger-Muller tube (04marks)

(c) A radioactive isotope $^{32}_{15}P$ which has a half-life of 14.3 days, disintegrates to form a stable product. A sample of the isotope is prepared with an initial activity of $2.0x10^{6}s^{-1}$. Calculate the,

(i) Number of $^{32}_{15}P$ atoms initially present **An[3.57x10^{12}atoms]** (03marks)

(ii) Activity after 30 days **An[4.67x10^{5}s^{-1}]** (03marks)

(iii) Number of $^{32}_{15}P$ atoms after 30 days **An[8.33x10^{11}atoms]** (02marks)

(Assume$N = N_0 e^{-\lambda t}$)

UNEB 2011 Q10

a) What is meant by unified atomic mass unit (1 mark)

b) (i) Distinguish between nuclear fission and nuclear fusion (2 marks)

ii) State the condition necessary for each of the nuclear reactions in b(i) to occur

c) (i) With the aid of a labeled diagram, describe the operation of an ionization chamber (6 marks)

ii)Sketch the curve of ionization current against applied p.d and explain its main features(4 marks)

d) A typical nuclear reaction is given by $^{235}_{92}U + ^{1}_{0}n \rightarrow \ ^{95}_{42}Mo + \ ^{139}_{57}La + 2^{1}_{0}n + 7^{\ 0}_{-1}e$

Calculate the total energy released by 1g of uranium

mass of $^{1}_{0}n = 1.009u$, of $^{\ 0}_{-1}e = 0.00055u$, $^{95}_{92}Mo = 94.906u$, $of\ ^{139}_{57}La = 138.906u$

$^{235}_{92}U = 235.044u.$ $1u = 1.66x10^{-27}kg$ **Ans** $[\mathbf{8.387x10^{10}J}]$

UNEB 2010 Q 10

a) (i) What is meant by mass defect? (1 mark)

(ii) Sketch a graph showing how binding energy per nucleon varies with mass number and explain its main features (3 marks)

iii)Find the binding energy per nucleon of $^{56}_{26}Fe$ given that mass of 1proton = 1.007825U. Mass of 1neutron=1.008665U, [1U = 931MeV] **[Ans 7.7Mev]**

b) With the aid of a diagram, explain how an ionization chamber works (6 marks)

UNEB 2003 Q10

a) What is meant by the following terms

i)Nuclear number

ii)Binding energy (2mk)

a) Calculate the energy released during the decay of $^{220}_{86}Ra$ nucleus into $^{216}_{84}Po$ and an alpha-particle

Mass of $^{220}_{86}Ra = 219.964176u$, Mass of $^{216}_{84}Po = 215.955794u$,Mass of $^{4}_{2}He = 4.001566u$

$(1u = 931MeV)$ **Ans [6.35MeV]**

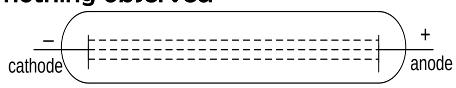
CHAPTER 4: CHARGED PARTICLES

4.1:CATHODE RAYS

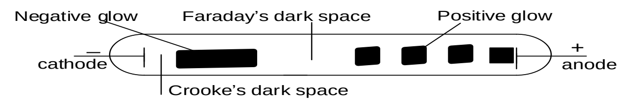
These are streams of fast moving electrons that travel from cathode to anode when a $p.d$ is connected across the plate.

4.1.0: Production of cathode rays by discharge tube method.

- At atmospheric pressure, the tube is clear with nothing observed



- At a pressure of 100 mmHg, streamers of luminous gas appear between the electrodes. Between 10 mmHg and 0.1 mmHg, the discharge becomes a steady glow spreading through the tube.



- Four regions form with the positive column occupying the larger part of the tube. The positive column forms striations when pressure is reduced further, the dark spaces swell and positive column shrinks
- At 0.1 mmHg, Crookes' dark space becomes distinct and the cathode glow appear round the cathode
- At 0.01mmHg, Crookes' dark space fills the glass tube and the tube fluoresces due to electron movement.

Limitations of the discharge tube method

- When cathode rays strike the anode they may produce x-rays which are dangerous
- A very high $p.d$ is needed across the electrodes which can be hazardous to handle
- The gas is needed at appropriate low gas pressure which is difficult to attain

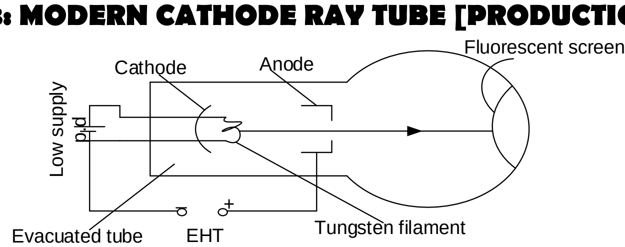
4.1.1:THERMIONIC EMISSION

- **Thermionic emission** is a process by which electrons are emitted from a hot metal surface.

4.1.2: MECHANISM OF THERMIONIC EMISSION

- Metals contain free electrons in their lattice that are loosely bound to their parent nuclei.
- As the temperature of the metal is raised, velocities of the electrons increase, some of the surface electrons acquire sufficient kinetic energy to overcome the electrostatics attraction force of the atomic nucleic and consequently escape from the metal surface.

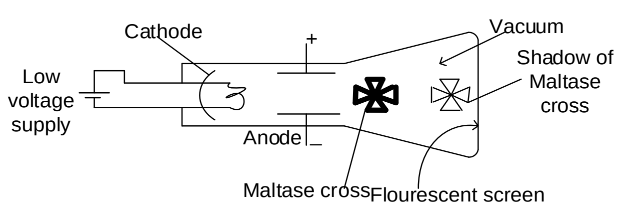
4.1.3: MODERN CATHODE RAY TUBE [PRODUCTION OF CATHODE RAYS]



- The cathode is heated by a low p.d and produces electrons by thermionic emissions.
- The electrons are focused by the cathode and accelerated by EHT to the fluorescent screen which gives a glow when they strike the screen.
- It is the beam of fast moving electrons from the cathode which constitute the cathode rays.

4.1.5:TO STUDY PROPERTIES OF CATHODE RAYS

1: Straight line movement



- Electrons emitted from a heated cathode, are accelerated by the anode and directed towards a maltase cross placed in the center of the a glass tube.
- A sharp shadow of the maltose cross is cast on a screen at the end of the tube. This shows that cathode rays travel in a straight line

4.1.6: Electron dynamics (motion of an electron)

(i) Motion in an Electric field

When an electron moves horizontally into a uniform electric field, it describes a parabolic path. This parabolic motion is brought by the electric force [F=Ee] experienced by electrons in the direction of that of the field.

Note

The horizontal motion of the electron is not affected by the field. A charge gains energy when it moves in the direction of an electric field and after leaving the plate the electron moves in a straight line

a) Speed of an electron

Suppose an electron of charge e and mass m is emitted from a hot cathode and **accelerated** by an electric field of potential V_a towards the anode, then;

Kinetic energy gained by the electron =work done on an electron by the accelerating p.d V_a

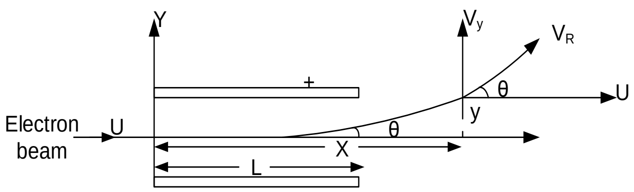
$$\tfrac{1}{2}mu^2 = e\,V_a$$

Or $$\boxed{u = \sqrt{\frac{2eV_a}{m}}}$$

Note V_a must be accelerating $p.d$ and not $p.d$ between the plates

b) Displacement of an electron in an electric field

Consider an electron of charge e and mass m entering an electric field horizontally with a speed u.



Force on the electron; F= $Ee = \frac{V}{d}e$------------[1]

Where E is electric field intensity, E= $\frac{V}{d}$

V- $p.d$ between the plates

d- distance of separation of plates

By Newton's 2nd law; F=ma ------------------- [2]

Equating 1 and 2 Ma= $\frac{V}{d}e$

Put into equation 3: a= $\frac{Ve}{md}$ ------------ [4]

Using $s = ut + {}^1\!/_2\,at^2$

(↑): [u=0m/s], y= ${}^1\!/_2\,\frac{Ve}{md}t^2$ -------- [5]

(→): [a=0m/s²], t= $\frac{x}{u}$ ------------------- [6]

put into equation 5: $y = {}^1\!/_2\,\frac{Ve}{md}\left(\frac{x}{u}\right)^2$

$$\boxed{\boldsymbol{y = \left(\frac{Ve}{2mdu^2}\right)x^2}}$$

Since $\left(\frac{Ve}{2mdu^2}\right)$ is constant, then $y \propto x^2$ then the motion is parabolic

c) Velocity of an electron in an electric field

Using $v = u + at$

(↑): [u=0m/s], $V_y = at$

$$V_y = \frac{Vex}{mdu}$$

d) Formula when the electron just leaves the plate

Just an electron leaves the plate $x=l$

$$\boxed{y = \left(\frac{Vel^2}{2mdu^2}\right)} \quad or \quad \boxed{y = \left(\frac{Ee}{2mu^2}\right)l^2}$$

(↑): [u=0m/s], $V_y = \frac{Vel}{mdu}$ or $V_y = \frac{Eel}{mu}$

Velocity with which the electron leaves the plate:

$$\boxed{V_R = \sqrt{v_y^2 + u^2}}$$

Direction with which the electron emerges :

$$\theta = tan^{-1}\left(\frac{V_y}{u}\right)$$

Examples

1. An electron is accelerated from rest through a p.d of 1000V. what is;
   (a) Its kinetic energy in eV
   (b) Its kinetic energy in joules
   (c) Its speed

Solution

a) $Va = 1000V: \Rightarrow \quad k.e = 1000eV$

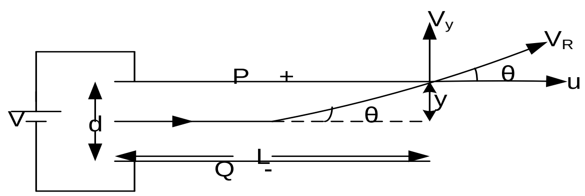
b) $k.e = eVa = 1.6x10^{-19}x1000 = 1.6x10^{-16}J$

c) $½ \text{ mu}^2 = eVa$

$u = \sqrt{\frac{2eV_a}{m}} = \sqrt{\frac{2x1.6x10^{-19}x1000}{9.11x10^{-31}}}$ =1.874x10⁷m/s

2. A beam of electrons, moving with velocity of $1.0x10^7ms^{-1}$, enters midway between two horizontal parallel plates P, Q in a direction parallel to the plates. P and Q are 5cm long, 2cm apart and have a $p.d$ V applied between them. Calculate V if the beam is deflected so that it just grazes the edge of the upper plate P

Solution



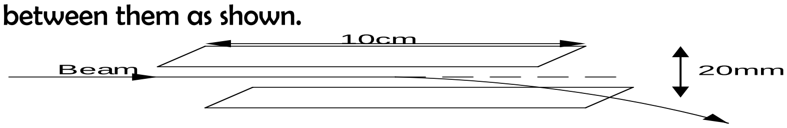
$y = \frac{d}{2} = 1cm$ since the beam is directed midway

$y = \frac{Vel^2}{2mdu^2}$

$0.01 = \frac{vx1.6x10^{-19}x}{2x9.11x10^{-31}\left(\frac{2}{100}\right)}\left(\frac{5}{100}\right)^2 x(1x10^7)^2$

$V = 91.1V$

3. Two parallel metal sheep's of length 10cm are separated by 20mm in a vacuum. A narrow beam of electrons enters symmetrically between them as shown.

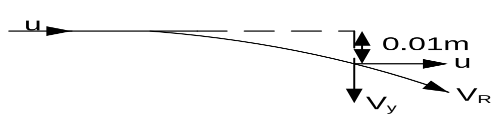


When a $p.d$ of 1000V is applied between the plates the electron beam just misses one of the plates as it emerges. Calculate the speed of the electrons as they enter the gap [Take the field between the plates to be uniform] [$^e/_m$ for the electron $= 1.8x10^{11}Ckg^{-1}$]

Solution

Since the beam enters symmetrically $y = \frac{d}{2} = \frac{0.02}{2} = 0.01m$

$d = 0.02m, L = 0.1m, V = 1000V$



but specific charge $\frac{e}{m}$ =1.8x10¹¹Ckg⁻¹

using y= $\frac{vet^2}{2md}$

when the beam just emerges $t = \frac{l}{u}$

$0.01 = \frac{1000x(0.1)^2}{2x0.02xu^2} x\left(\frac{e}{m}\right)$

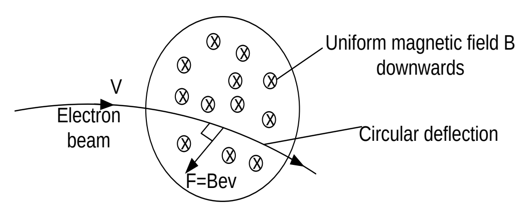
$0.01 = \frac{1000x(0.1)^2x1.8x10^{11}}{2x0.02xu^2}$

$u^2 = 4.5x10^{15}$

$u = 6.71x10^7ms^{-1}$

(ii) Motion is a magnetic field

When an electron beam having a common velocity enters a uniform magnetic field, the electrons experience a constant magnetic force $F = BeV$ at right angles to both B and V according to Fleming left hand rule and the ion describes a circular path of radius r given by $\left(\frac{mv^2}{r} = BQv\right)$ hence $\boxed{r = \frac{mv}{BQ}}$



Examples

1. An electron is moving in a circular path at $3.0x10^6ms^{-1}$ in a uniform magnetic field of flux density $2.0x10^{-4}T$. Find the radius of the path [mass of electron=$9.1x10^{-31}$kg,charge on electron= $1.6x10^{-19}$C]

Solution

$\therefore \frac{mv^2}{r} = BQv$ | $r = \frac{mv}{BQ}$ | $r = \frac{9.1x10^{-31}x3x10^{6}}{2x10^{-4}x1.6x10^{-19}} = 0.0853m$

2. Electrons accelerated from rest through a potential difference of 3000V enters a region of uniform magnetic field, the direction of the field being at right angles to the motion of the electrons. If the flux density is 0.01T, calculate the radius of the electron orbit. [Assume that the specific charge e/m for the electron $=1.8x10^{11}Ckg^{-1}$]

Solution

4.2.0:POSITIVE RAYS

These are streams of positively charge particles that pass through a perforated cathode

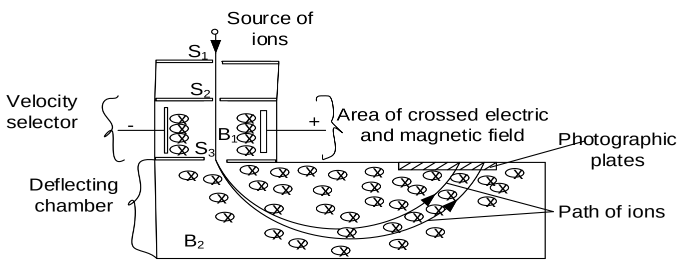
4.2.1: Production of positve rays

- Positive rays are produced when cathode rays in a discharge tube collide with gaseous atoms and strip off (knock out) some electrons from the atoms.
- The positive ions formed are accelerated to the cathode and these streams of positive ions constitute rays.

4.3.0: SPECIFIC CHARGE OF AN ION

This is the ratio of charge to mass of an ion. S.I unit is $C\ kg^{-1}$

4.3.1: Determination of the specific charge of ions using a Bain bridge mass spectrometer



- Streams of ions from a source is directed through slits S_1 and S_2 into the velocity selector where there are crossed electric field of intensity, E and magnetic field of flux density, B_1
- Ions of charge, Q pass through the selector un deflected with velocity, u given by $B_1Qu = EQ$, that is

$$u = \frac{E}{B_1}$$

- The selected ions pass through S_3 and enter a deflection chamber with a uniform magnetic field of flux density, B_2
- The ions move along a semi circular path and strike the photographic plate where they are detected. The radius, r of the path described is measured and recorded.
- In a circular path, $B_2Qu = \frac{mu^2}{r}$, that is $\frac{Q}{m} = \frac{U}{B_2r}$
- On substituting for u, the charge to mass ratio is got from $\boxed{\frac{Q}{m} = \frac{E}{B_1B_2r}}$

Examples

1. A beam of protons is accelerated through a $p.d$ of 10kV and is allowed to enter a uniform magnetic field B of 0.5T perpendicular to their path. Find the radius of the circle they travel. [$mass\ of\ proton = 1.67x10^{-27}kg$, $e = 1.6x10^{-19}C$]

Solution

$$u = \sqrt{\frac{2eVa}{m}}$$

$$u = \sqrt{\frac{2x1.6x10^{-19}x10x10^{3}}{1.67x10^{-27}}} = 1.38x10^{6}ms^{-1}$$

In the magnetic field : $\frac{mu^2}{r} = Beu$

$$r = \frac{mu}{Be} = \frac{1.6x10^{-27}x1.38x10^{6}}{0.5x1.6x10^{-19}} = 0.029m$$

2. In a Bain bridge mass spectrometer singly ionized atoms of ^{35}Cl, ^{37}Cl pass into the deflection chamber with a velocity of $10^5 ms^{-1}$. If the flux density of the magnetic field in the deflecting chamber is 0.08T, calculate the difference in the radii of the path of the ion. [$1u = 1.67x10^{-27}kg$, $e = 1.6x10^{-19}C$]

Solution

Let r_1 be radius for ^{35}Cl and r_2 be radius for ^{37}Cl

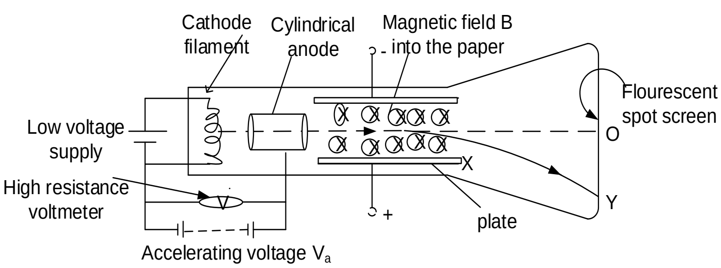
$$1u = 1.66x10^{-27}kg$$
$$35u = (1.66x10^{-27}x35)kg$$
$$37u = (1.66x10^{-27}x37)kg$$
$$\frac{mu^2}{r} = Beu$$

$$r_1 = \frac{35x1.66x10^{-27}x10^5}{0.08x1.6x10^{-19}} = 0.454m$$
$$r_2 = \frac{35x1.66x10^{-27}x10^5}{0.08x1.6x10^{-19}} = 0.480m$$

Difference $r_2 - r_1 = 0.48 - 0.454 = 0.026m$

4.3.2: DETERMINATION OF SPECIFIC CHARGE OF AN ELECTRON BY J.J THOMSON'S EXPERIMENT

The charge per unit mass or specific charge of an electron can be measured by the apparatus below.



- ❖ Electrons are emitted thermionic ally by the filament and are accelerated towards the cylindrical anode.
- ❖ With no electric and no magnetic fields applied at plate X, the electron beam strikes the screen at point O which is noted
- ❖ separation

Examples

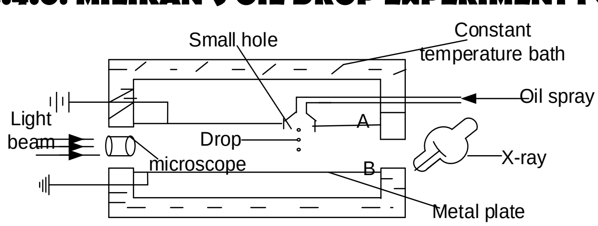
1. Two plates are 2cm long and separated by a distance of 0.5cm in a uniform magnetic field of flux density $4.7x10^{-3}T$. An electron beam incident midway between the plates is deflected by magnetic field through a distance of 10cm on a screen placed 24cm from the end of the plate. When a $p.d$ of 1000V is applied to the plate , the electron is restored to the un deflected position. Calculate the specific charge of the electron

Solution

2. An electron beam in which the electrons are $2x10^7 ms^{-1}$ enters a magnetic field in a direction perpendicularly to the field direction. It is found that the beam can pass through without change of speed or direction. When an electric field of strength $2.2x10^4 Vm^{-1}$ is applied in the same region at a suitable orientation. [$e=1.6x10^{-19}C$]
   (i) Calculate the strength of the magnetic field
   (ii) If the electric field were switched off, what would be the radius of curvature of the electron path.

Solution

4.4.0: MILIKAN'S OIL DROP EXPERIMENT FOR MEASUREMENT OF CHARGE



- ❖ Oil is sprayed above the upper metal plate.
- ❖ With no P.d between the plates, one oil drop is observed as it falls between the plates.
- ❖ The distance, x fallen in time, t is obtained and te1rminal velocity V_t of the drop is determined.

At terminal velocity: $\frac{4}{3}x\pi r^3\rho_o g = \frac{4}{3}\pi r^3\rho_a g + 6\pi\eta r V_t$

$$r = \left[\frac{9\eta v_t}{2g(\rho_o - \rho_a)}\right]^{\frac{1}{2}}$$

- ❖ A P.d is applied across the plates and is adjusted until the drop becomes stationary. P.d V and separation d between plates are measured and recorded, $E = \frac{V}{d}$ is calculated

$$\frac{4}{3}x\pi r^3\rho_o g = \frac{4}{3}\pi r^3\rho_a g + EQ$$

$$Q = \frac{6\pi\eta V_t}{E}\left[\frac{9\eta V_t}{2g(\rho_o - \rho_a)}\right]^{\frac{1}{2}}$$

ρ_o is density of oil

ρ_a is density of air

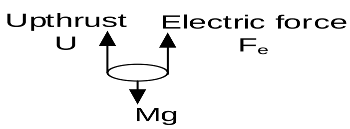
η viscosity of air

❖ Using several drops, the charge on each drop is obtained. The charge on each drop is an integral multiple of e which is the electron charge

Examples

1. In Millikan's experiment an oil drop of mass $1.92x10^{-14}$kg is stationary in the space between the two horizontal plates which are $2x10^{-2}$m apart, the upper plate being earthed and the lower one at a potential of -6000V. Neglecting the buoyancy of the air. Calculate the magnitude of the charge.

Solution



At terminal velocity: $Mg = U + F_e$

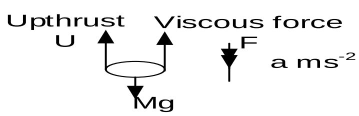
But u=0 [neglecting air buoyancy]: $EQ = mg$

$Q = \frac{mgd}{v}$ since $E = v/d$

$Q = \frac{1.92x10^{-14}x9.81x2x10^{-2}}{6000}$ = $6.28x10^{-19}$C

2. Calculate the radius of drop of oil of density 900kgm^{-3} which falls with a terminal velocity of $2.9x10^{-2}$ms^{-1} through air of viscosity $1.8x10^{-5}$Nsm^{-2}. Ignore the density of air, if the charge on the drop is -3e. What p.d must be applied between two plates 5cm apart for the drop to be held stationary between them [$e=1.6x10^{-19}$C]

Solution

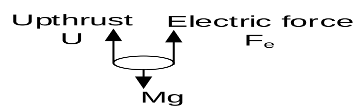


u=0 and at terminal velocity: $F = mg$

$6\pi\eta r v_t = \frac{4}{3}\pi r^3\rho_o g$

$$r = \sqrt{\frac{9\eta v_t}{2g\rho_o}} = \sqrt{\frac{9x1.8x10^{-5}x2.9x10^{-2}}{2x900x9.81}} = 1.63x10^{-5}\text{m}$$

For the drop to be held stationary then there is no viscous drag



u=0 and at terminal velocity: $mg = EQ$

$E = {}^{V}/_{d}$ and $Q = 3e$

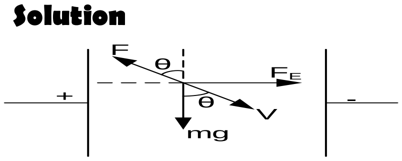
$$V = \frac{mgd}{Q} = \frac{\frac{4}{3}\pi r^3\rho_o g}{3Q}$$

$$V = \frac{4x\frac{22}{7}x(1.63x10^{-5})^3x900x9.81x5x10^{-2}}{3x3x1.6x10^{-19}}$$

$V = 1.67x10^7$V

3. An oil drop of mas $3.25x10^{-12}g$ falls vertically with uniform velocity through the air between parallel plates which are 2cm apart. When a p.d of 1kV is applied to the plates, the drop moves towards the negatively charged plate, its path being inclined at 45° to the vertical. Explain why the vertical component of its velocity remains unchanged and find the charge on the drop

Solution



The drop falls steadily due to viscosity of the air since the electric force is horizontal and has no (↑) component

$$(\rightarrow): Fsin\theta = EQ = \frac{VQ}{d} \ldots\ldots\ldots\ldots\ldots\ldots 1$$

$$(\uparrow): Fcos\theta = mg \ldots\ldots\ldots\ldots\ldots 2$$

$$2 \div 1 \qquad tan\theta = \frac{vQ}{mgd}$$

$$Q = \frac{mgd\ tan\theta}{v}$$

$$Q = \frac{3.25x10^{-15}x9.81x0.02x\ tan45}{1000}$$

$Q = 6.38x10^{19}C$

Exercise :60

1. A spherical oil drop of radius of $2x10^{-6}$m is held stationery between two parallel metal plates to which a $p.d$ of 4500V is applied , the separation of the plates is 1.5cm, calculate the charge on the drop if the density of oil is 800kgm^{-3}. Assume no air resistance. **An**[$8.76x10^{-19}$ C]
2. In milikans oil drop experiments, a charged droplet falls with a velocity of $0.04mm\ s^{-1}$ when no voltage is applied to the plates. The same drop can be held stationary between the plates when a voltage of 23.7V is applied between them. If the drop has a diameter of 1 mm and the plate are 10mm apart. Calculate
   (i) Charge on the drop
   (ii) New velocity of the drop when a potential difference of 50V is applied between the plates
   An[$2.86x10^{-15}$ C, $4.4x10^{-5}ms^{-2}$]

UNEB 2020 Q.10

(a) Define **Specific charge** of a positive ion and state its unit (02marks)

(b) With the aid of a labelled diagram, describe Bainbridge spectrometer can be used to determine the specific charge of positive ions. (06marks)

(c) A beam of positive ions accelerated through a potential difference of 2,000V enters a region of uniform magnetic flux density B. The ions describe a circular path of radius 3.2cm while in the field. If the specific charge of the ions is $8.5x10^{7}Ckg^{-1}$, derive an expression for the charge to mass ratio of the ions and use it to calculate the value of B **An(0.214T)** (05marks)

(d) State the use of each of the following features of Cathode Ray Oscilloscope (C.R.O)
   (i) Anode system (01mark)
   (ii) Y-plates (01mark)
   (iii) The grid (01mark)

(e) An electron with energy 5keV moves in the direction of an electric field of internsity $1.6x10^{4}Vm^{-1}$. What distance will the elctron move before coming to rest **An(0.3125m)** (04marks)

UNEB 2019 Q.8

(a) (i) What are **cathode rays?** (01mark)
   (ii) State **two** properties of cathode rays (01mark)
   (iii) Explain **two** disadvantages of using the distube in producing cathode rays (02marks)

(b) With the use of a labelled diagram, describe Milikan's experiment to determe charge on an oil drop (07marks)

(c) A beam of electrons is accelerated through a potential difference of 1.98kV and directed mid-way between two horizontal plates of length 4.8cm and separated by a distance of 2.0cm. The potential difference applied between the plats is 80.0V
   (f) Calculate the speed of the electrons as they enter the plates **An ($2.64x10^{7}ms^{-1}$)** (03marks)
   (g) Explain the motion of the electron between the plates (02marks)
   (h) Find the speed of the electron as they emerge from the region between the plates
   An ($2.643x10^{7}ms^{-1}$) (04marks)

UNEB 2018 Q.18

(a) Explain what is observed in a discharge tube when the pressure is gradually reduced to low values (05marks)

(b) With the aid of a labeled diagram, describe the operation of a Bainbridge spectrometer in the determination of charge to mass ratio. (07marks)

(c) An ion of mass $2.6x10^{-26}kg$ moving at a speed of $4.0x10^{4}ms^{-1}$ enters a region of uniform maganetic field of flux density 0.05T. Calculat the radius of the circle described by the ion. **An[0.13m]** (03marks)

UNEB 2013 Q.8

(a) Explain briefly how **positive rays** are produced (03marks)

(b) An electron of charge, e and mass, m, is emitted from a hot cathode and then accelerated by an electric field towards the anode. If the potential difference between the cathode and anode is V, show that the speed of the electron. U. is given by $u = \sqrt{\left(\frac{2\,e\,V}{m}\right)}$ (03marks)

(c) An electron starts from rest and moves in an electric field intensity of $2.4x10^{3}Vm^{-1}$. Find the;

(i) Force on the electron. **An** $(3.84x10^{-16}N)$ (02marks)

(ii) Acceleration of the electron **An** $(4.22x10^{14}ms^{-2})$ (02marks)

(iii) Velocity acquired in moving through a $p.d$ of 90V **An** $(5.62x10^{6}ms^{-1})$ (02marks)

(d) A beam of electron each of mass, m, and charge, e, is directed horizontal metal plates separated by a distance, d.

(i) If the $p.d$ between the plates is V, show that the deflection y of the beam is given by

$$y = \frac{1}{2\,m}\left(\frac{e\,V}{d\,u^{2}}\right)x^{2}$$

Where, x, is the horizontal distance travelled (06marks)

(ii) Explain the path of the electron beam as it emerges out of the electric field (02marks)

UNEB 2010 Q.8

(a) (i) With the aid of a labeled diagram, describe what is observed when a high tension voltage is applied across a gas tube in which pressure is gradually reduce to very low values (05marks)

(ii) Give two applications of a discharge tubes (01mark)

(b) Describe Thomson's experiment to determine the specific charge of an electron (06marks)

(c) In Millikan's oil drop experiment, a charged oil drop of radius $9.2x10^{-7}m$ and density 800 kgm^{-3} is held stationary in an electric field of intensity $4x10^{4}Vm^{-1}$.

(i) How many electron charges are on the drop [04marks]

(ii) Find the electric field intensity that can be applied to move the drop with velocity $0.005ms^{-1}$ upwards (density of air $=1.29kgm^{-3}$, $\eta=1.8x10^{-5}Nsm^{-1}$) [04marks] **An[4, $2.48x10^{6}$ Vm^{-1}]**

UNEB 2003 Q.8

(b) Explain how Millikan's experiment for measuring the charge of the electron proves that the charge is quantized.

(c) A beam of positive ions is accelerated through a $p.d$ of 1000V into a region of uniform magnetic field of flux density 0.2T. While in the magnetic field it moves in a circle of radius 2.3cm. Derive an expression for the charge to mass ratio of the ions and calculate its value. **An[$9.45x10^{7}Ckg^{-1}$]**

UNEB 2002 Q.9

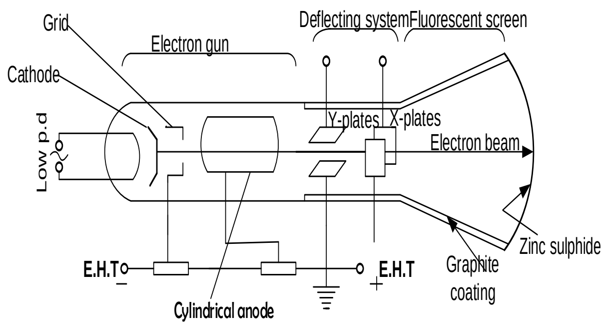
(a) (i) What are cathode rays? [01mark]

(ii) An electron gun operating at $3x10^{3}V$ is used to project electrons into the space between two oppositely charged parallel plates of length 10cm and separation 5cm, calculate the deflection of the electrons as they emerge from the region between the charged plates when the $p.d$ is 1000V.

An[$1.66x10^{2}m$] [04marks]

CHAPTER 5: ELECTRONIC DEVICES

5.1.0: THE CATHODE RAY OSCILLOSCOPE (CRO)



- ❖ Cathode is heated and emits electrons thermionic ally. The electrons are focused and accelerated by the anodes to the screen. Grid controls number of electrons reaching the screen hence brightness of the spot
- ❖ Y-plates deflect electron beam vertically and X-plates deflect electron beam horizontally.
- ❖ The screen glows to form a spot when struck by electrons. Graphite coating shields electrons from external fields and conducts stray electrons to the earth.

USES OF THE CRO

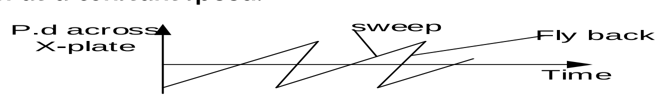
- ❖ It is used to display wave forms
- ❖ It measures voltage (AC or DC)
- ❖ Measures frequencies
- ❖ Used to measure phase differences
- ❖ Measures small time intervals

Advantages of CRO over a voltmeter

- ❖ It measures both AC and D.C voltage unlike a voltmeter measures only D.C voltage unless a rectifier is used
- ❖ It has an instantaneous response since the electron beam behaves as a pointer of negligible inertia.
- ❖ It draws very little current since it has nearly infinite resistance to DC and a very high impedance to AC
- ❖ It has no coil to burn out.

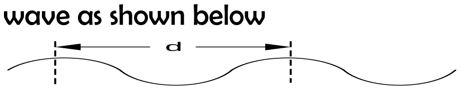
Time base

- This is a circuit connected to the x-plates of a C.R.O and provides a saw tooth $p.d$ that sweeps the electron beam across the screen at a constant speed.



Measurement of the frequency of an A.C signal using a C.R.O

- ❖ The time base is set at $Xmscm^{-1}$
- ❖ A signal is applied on the Y-plate to obtain a wave as shown below



- ❖ The distance, d between successive crests is measured and recorded
- ❖ The period of the wave , $T = (Xgain)x10^{\ -3}xd$
- ❖ The frequency of the wave, $f\ = \frac{1}{T}$

Examples

1. If the voltage gain is 20Vcm^{-1} and an alternating voltage connected to Y-plate products a vertical trace of 12cm long with time base off. Find the peak value of the voltage and its r.m.s value

Solution

$2V_0 = V_gL$ | $V_0 = \dfrac{20x12}{2} = 120V$ | Vr.m.s $= \frac{V_0}{\sqrt{2}} = \frac{120}{\sqrt{2}} = 84.85V$

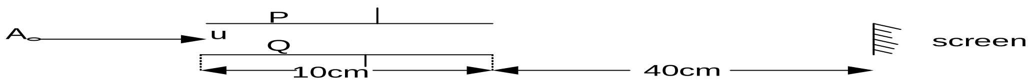
2. An alternating p.d applied to the Y-plate of an oscilloscope produces five complete waves on a 10 cm length of the screen when the time base setting is $10ms\ cm^{-1}$. Find the frequency of the alternating voltage.

Solution

$$T = \frac{(Xgain)x10^{-3}xd}{5} = \frac{10x10^{-3}x10}{5}$$
$$T = 0.02s$$

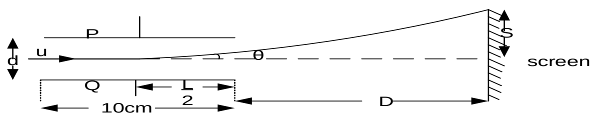
$$f = \frac{1}{T} = \frac{1}{0.02}$$
$$f = 50Hz$$

3. The sketch below shows part of the deflecting system of a cathode ray oscilloscope. At the point A, a beam of electrons has a velocity of $3x10^7ms^{-1}$ along the axis of the system. The plates which are 4cm apart provides a uniform electric field in the space between them. Edge effects may be neglected, P is at a potential of +200V with respect to Q



Find the position at which the electron beam strikes the screen ($^e/_m = 1.76x10^{11}Ckg^{-1}$)

Solution



L=$10x10^{-2}$m, d=$4x10^{-2}$m, D=$40x10^{-2}$m, V=200,

Tan $\theta = \frac{S}{D+\frac{l}{2}}$ ------------------------- [1]

But also $Tan\ \theta = \frac{v_y}{u}$ ------------------- [2]

Equating 1 and 2: $\frac{S}{D+\frac{l}{2}} = \frac{v_y}{u}$

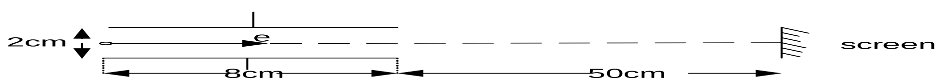
S= $\frac{v_y}{u}(D + \frac{l}{2})$

But V_y= $\frac{vel}{mdu^2}$ $(D + \frac{l}{2})$

$$S = \frac{200x1.76x10^{11}x10x10^{-2}x}{4x10^{-2}x(3x10^7)^2} \text{x } [4x10^{-2} + \frac{10x10^{-2}}{2}]$$

$$S = 0.044m$$

4. The figure below shows two metal plates 8cm long and 2cm apart. A fluorescence screen is placed 50cm from the one end of the plates. An electron of kinetic energy $6.4x10^{-16}J$ is incident midway between the plates



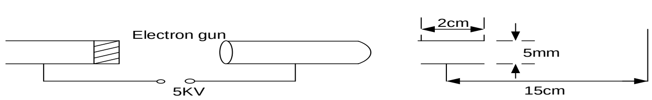
Calculate the $p.d$ which must be applied across the plates to deflect the electron 4.2cm on the screen. Assume that the space through which to electron moves is evacuated. $[e = 1.6x10^{-19}C,\ m = 9.1x10^{-31}kg$

Solution

$$V = 156V$$

Exercise: 61

1.



Calculate the deflection sensitivity (deflection of spot in mm per volt potential difference) of the cathode ray tube from the following data.
Electrons are accelerated by a potential difference of 5kV between the cathode and anode. [length of deflection plates =2cm, separation of deflector plates =5mm, distance of mid point of deflector plates from screen =15cm] **An [$6x10^{-2}mmV^{-1}$]**

(i) Deflection of electron beam on the screen.

3. A C.R.O consists of two metal plates 3.5cm long and 2.5cm apart with the upper plate being positive. An electron is projected along the axis of a C.R.O at a velocity of $1.5x10^7\ ms^{-1}$ in a uniform electric field of $3x10^4\ Vm^{-1}$. Calculate;
   (i) How far above the axis the electron will be when its leaves the space between the plates
   (ii) How far above the axis the electron beam will strike the screen, if the fluorescence screen is placed 15cm from the one end of the plates
   An[1.07, 11.7cm]

UNEB 2020Q.9

(d) (i) Explain the of thermionic emission [03marks]

(ii) The gain control of a cathode ray oscilloscope is set at 0.5Vcm^{-1} and an alternating voltage produced a a vertical line of length 2.0cm with time base off. Find the root mean square value of potential difference. **An(0.354V)** [02marks]

UNEB 2019 Q.9

(b) (i) Draw a well labeled diagram to show the main parts of a C.R.O [03marks]

(ii) Describe how a C.R.O can be used as an a.c voltmeter [02marks]

UNEB 2016Q.10

(c)(i) What is a **time base** as applied to a cathode ray oscilloscope. (01mark)

(ii) Draw a sketch graph showing the variation of time base voltage with time. (01mark)

(a) An alternating p.d applied to the Y-plate of an oscilloscope produces five complete waves on a 10 cm length of the screen when the time base setting is $10ms\ cm^{-1}$. Find the frequency of the alternating voltage. **An(50Hz)**

UNEB 2011 Q.8

(a) (i) Describe with the aid of a well labeled diagram, the structure and mode of operation of CRO

(ii) State the advantages of CRO over a moving coil voltmeter [02marks]

UNEB 2004 Q.8

(a) (i) Describe with the aid of a labeled diagram the main features of a cathode ray oscilloscope (CRO)

(ii) State two uses of a CRO [01marks]

(iii) The gain control of a CRO is set on 0.5Vcm^{-1} and an alternating voltage produces a vertical trace of 2cm along with the time base off. Find the root mean square value of the applied voltage. **An[0.354V**]

UNEB 2005 Q.9

(b) Describe, with the aid of a diagram, the structure and mode of operation of a cathode ray oscilloscope (CRO) [06marks]

(c) A CRO has its y-sensitivity set to 20Vcm^{-1}, a sinusoidal input voltage is suitably applied to give a steady time base switched on so that the electron beam takes 0.01s to traverse the screen. If the trace seen has a peak – to-peak height of 4cm and contains two complete cycles. Find the

(i) r.m.s value of the input voltage [03marks]

(ii) frequency of the input signal **An[14.14V, 200Hz]** [02marks]

5.2.0: SEMICONDUCTORS

Semiconductors are materias whose electrical conductivities are higher than those of insulators but less than those of conductors.

Commonly used semiconducting materials include silicon, germanium, carbon and gallium arsenide.

Intrinsic semiconductors

This is a pure semiconductors with nothing added to it.

In the intrinsic semiconductor at very low temperatures, all the valence electrons are involved in boding, and the crystal is a perfect insulator because there are no electrons available for conduction. At higher temperatures some of the valence elctrons have sufficient energy to break away from the bonds and move about the structure. The higher temperature, the greater the number of free electrons, hence semiconductors have a negative temperature coefficients of resistance, i.e their electrical resistivities decrease with increasing temperature.

When an electron jumps into a conduction band it leaves behind it a space or **a hole** in the valence band. This hole is effectively positive and since an electron can jump into it from from another part of the valence band, it is as if the hole itself was moving. Conduction can take place either by electrons moving within the conduction band or by positive holes moving within the valence band.

Extrinsic semiconductors

Extrinsic Semiconductors is a semiconductor to which a very small amount of impurity has been added by a procces called **doping.**

The extent to which a semiconductor conducts electricity is considerably affected by the presence of impurities.

Doping

Doping is the introduction of controlled amounts of pentavalent materials into one half of a group IV semiconductor and trivalent materials on the other half of a group IV semiconductor. The first half has electrons as the majority charge carriers and therefore called n-type while the second half has holes as the majority charge carriers forming the p-type.

Types of extrinsic semiconductors

(i) **n-type**

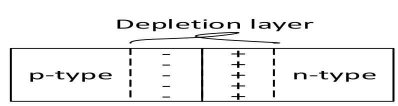
This a semiconductor in which electrons are majority carriers. It is made by doping with a pentavalent material such as phosphorus.

(ii) **p-type**

This a semiconductor in which holes are majority carriers. It is made by doping with a trivalent material such as alluminium

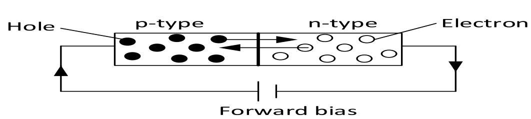
5.2.1: THE p-n JUNCTION DIODE

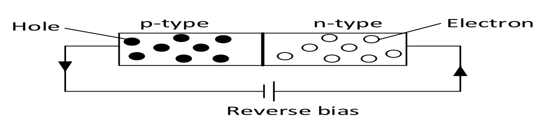
A p-n junction is formed by melting the boundaries of a p-type and n-type semiconductors and joining them. At the boundary, holes drift from the p-type towards the n-type material and at the same time electrons drift from the n-type to the p-type. The diffusion of holes and electrons across the boundary (depletion layer) sets up a potential barrier which prevents further change, the p-type region becoming slightly negative and the n-type becoming slightly positive.



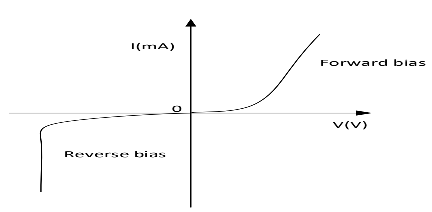
It is the existence of the junction between the two types of semiconducting material which gives the device its ability to rectify.

When a rectifier is connected to a supply, its supposed to conduct and when it does so its said to be **forward biased**. And when connected in a reverse way it fails to conduct therefore its said to be **reverse-biased.**





5.2.2: Characteristics of a p-n junction diode



- When in forward biased direction, as the p.d across it is increased, there is a large flow of electrons and current increases almost linearly.
- In reverse bias direction, the current is very small due minority charge carriers flow until the diode breaks down

5.2.3: Advantages of semiconductor diodes over thermionic diode

- They require less voltage to operate
- Semiconductor diodes do not waste much energy as heat
- They are quick and cheap to make

- ❖ They are small and portal

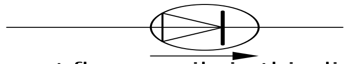
5..3.1: RECTIFICATION

Rectification involves converting Alternating current to Direct current.
This can be done by use of

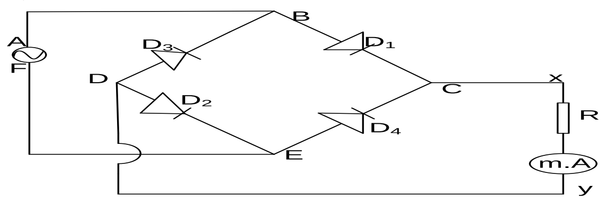
- ❖ Thermionic diodes.
- ❖ Semiconductor diode

The junction diode has low resistance to current flow when forward biased, but has high resistance to current flow when it is reverse biased.

Circuit symbol



a) Full wave rectification



- ➢ Four diodes are arranged in a bridge network as shown above. If A is positive during the first half cycle, diodes 1 and 2 conduct and current takes the path ABCRDEF
- ➢ During the next half cycle when F is positive and A is negative diodes D_3 and D_4 conduct while D_1 and D_2 do not conduct in this cycle and current (I) flows through path FECRDBA. The current through R is in the same direction throughout and it can be measured by moving coil ammeter.

5.4.0: THE JUNCTION TRANSISTOR (BIPOLAR TRANSISTOR)

A junction transistor is a single crystal of semiconducting material doped in such a way that a piece of p-type material is sandwiched between two pieces of n-type material, or such that a piece of n-type is between two pieces of p-type .
The three regions of the junction transistor are called the emitter, the base and the collector

Types of junction transistors

i) n-p-n transistor. Current is mainly due to electrons flowing from emitter to collector.
ii) p-n-p transistor. Current is mainly due to movement of holes from emitter to collector.

Symbols

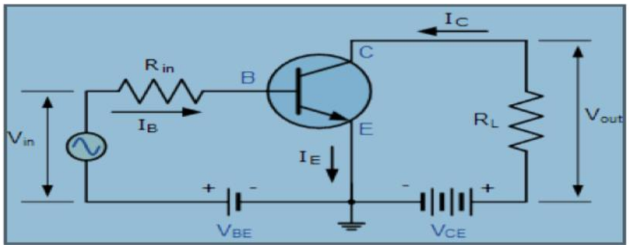
The arrows show direction of conventional current

The transister can be connected into a circuit in three different ways

- Common Emitter mode- has both Current and Voltage Gain.
- Common Base mode - has Voltage Gain but no Current Gain.
- Common Collector mode - has Current Gain but no Voltage Gain.

5.4.1:Common – emitter mode (CE mode) for n-p-n transistor

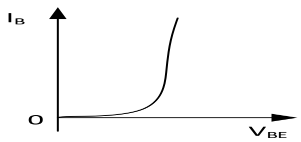
When a transistor is in use the base-emitter junction is normally forward biased and the base-collector junction is reverse biased. In the case of n-p-n transistor, the base must be positive with respect to the emitter and the collector must be positive with respect to the base



The circuit can be used to obtain three types of characteristics

(1) Input characteristics
(2) Out put characteristics
(3) Transfer characteristic

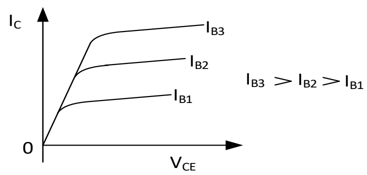
5.4.2: I_B Against V_{BE}(Input characteristics)



I_B varies exponentially with V_{BE}

Input resistance $R_{IN} = \frac{\Delta\ V_{BE}}{\Delta\ I_B}$

5.4.3: Collector current (I_C) Against collector emitter voltage (V_{CE}) (Output characteristics)

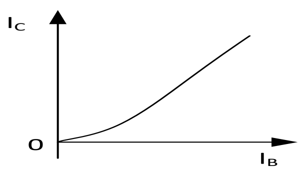


For small V_{CE} the output current I_C increases slightly with V_{CE}.

At Higher V_{CE}, I_C varies linearly with V_{CE} for a given base current I_B. the linear part of the characteristics is used as amplifier circuit so that the output voltage variation is undistorted.

Load resistance $R_L = \frac{\Delta\ V_{CE}}{\Delta\ I_C}$

5.4.4: A graph of I_C Against I_B (Transfer characteristics)

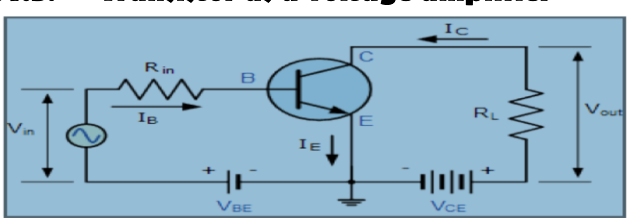


Output current I_C varies fairly linearly with the input current I_B.

Current transfer ratio βor (current gain)

$\beta = \frac{output\ current}{input\ current} = \frac{\Delta I_C}{\Delta I_B}$

5.4.5: Transistor as a voltage amplifier



The small A.C voltage V_{in} is applied to the base emitter circuit and causes a small change in base current I_B which produces a large change I_c in the collector current flowing through the load R

Numerical calculations

From the circuit diagram above, the current flowing out of the transistor must be equal to the currents flowing into the transistor

$$I_E = Ic + I_B.$$

Current gain is given by;

$$\beta = {}^{I_C}\!/_{I_B}$$

Also

$$V_{in} = I_B R_B + V_{BE}$$

$$V_{in} = I_C R_L + V_{CE}$$

Voltage amplification or voltage gain

This is the ratio of output voltage $V_0$to the input voltages V_{in}

$$\boxed{Voltage\ gain = \frac{V_0}{V_{in}}}$$

Input voltage; $V_{in} = I_B\ (R_{in} + r\ )$

R_{in}- input resistance of the base-emitter junction

r- internal resistance of the input source

$I_B\ = \frac{V_{in}}{R_{in}+r}$ ………………… [1]

$I_C = \beta I_B$ ……………. [2]

$I_C = \beta\left(\frac{V_{in}}{R_{in}+r}\right)$ ……………[3]

Output voltage; $V_o\ = I_C\ R_L$ ………… [4]

$$V_o\ = \beta\left(\frac{V_{in}}{R_{in}+r_s}\right)\ R_L$$

$$\frac{V_0}{V_{in}} = \frac{\beta\ R_L}{R_{in}+r}$$

$$\boxed{Voltage\ gain\ = \frac{\beta R_L}{R_{in}+r}}$$

Examples

1. An n-p-n Transistor has a DC current gain, (Beta) value of 200. Calculate the base current I_B required to switch a resistive load of 4mA.
Solution
$\beta = {}^{I_C}/_{I_B}$ | $I_B = {}^{4x10^{-3}}/_{200} = 20\mu A$
2. An n-p-n Transistor has a DC base-bias voltage of 10V and an input base resistance of 100$k\Omega$. Calculate the base current into the transistor if the base-emitter voltage drop is $0.7V$
Solution
$V_{IN}\ = I_B\,R_B +\ V_{BE}.$ | $10 = I_B\,x100{,}000 + 0.7$ | $I_B = 93\mu A$
3. The input resistance of a certain n-p-n transistor in the common emitter connection is 1$k\Omega$. The small current amplification transfer ratio is 100. The internal resistance of the emitter- base junction is negligible and the load resistor is 2.5 $k\Omega$. Find the voltage gain
Solution
$Voltage\ gain\ = \frac{\beta R_L}{R_{in}+r}$ | $Voltage\ gain\ = \frac{100x2500}{1000+0} = 250$

Trial exercise

1. An n-p-n Transistor has a DC base-bias voltage of 9V and base current of 20μA. Calculate the base resistance, if the base-emitter voltage drop is $0.8V$. **An** 410$k\Omega$
2. Determine the voltage amplification of a transistor with current amplification of 100, if the input resistance is 1$k\Omega$, the load resistor is 2.2 $k\Omega$ and the value of the resistor in the base circuit is 20$k\Omega$ **An 10.5**

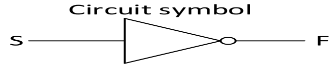
5.5.0: LOGIC GATES AND BOOLEAN ALGEBRA

Logic gates are the switches that turn ON or OFF depending on what the user is doing.
The output is either ON (1) or OFF (0) depending on the input.

NOT gate /INVERTOR gate

In the NOT gate, the output is high only when its input is not high

Circuit symbol



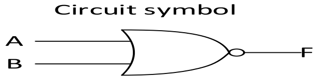
Truth table

Input S	Output F
0	1
1	0

NOR gate

It has two inputs and the output is high only when both inputs are not high

Circuit symbol



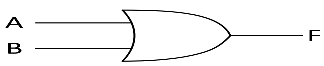
Truth table

Input A	B	Output F
0	0	1
0	1	0
1	0	0
1	1	0

OR gate

It has two inputs and the output is high if one input <u>or</u> both inputs are high

Circuit symbol



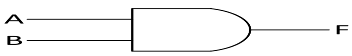
Truth table

Input		Output
A	B	F
0	0	0
0	1	1
1	0	1
1	1	1

AND gate

It has two inputs and the output is high only if one input is high <u>and</u> the other is also high

Circuit symbol



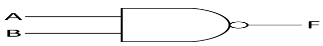
Truth table

Input		Output
A	B	F
0	0	0
0	1	0
1	0	0
1	1	1

NAND gate

It has two inputs and the output is high if one input is low <u>or</u> both are low

Circuit symbol

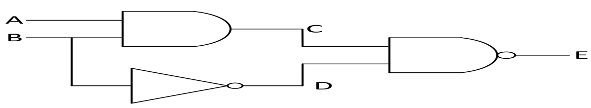


Truth table

Input		Output
A	B	F
0	0	1
0	1	1
1	0	1
1	1	0

Examples

1. Complete the truth table for the simple combination of logic gates below

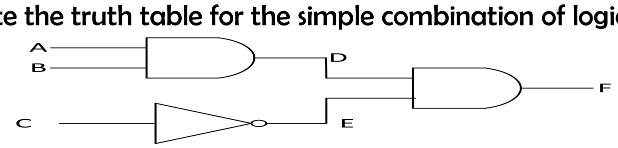


A	B	C	D	E
0	0			
0	1			
1	0			
1	1			

Solution

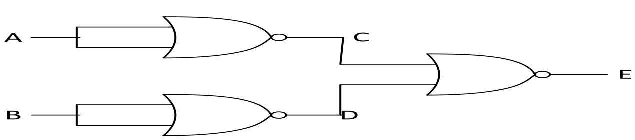
A	B	C	D	E
0	0	0	1	1
0	1	0	0	1
1	0	0	1	1
1	1	1	0	1

2. Complete the truth table for the simple combination of logic gates below



A	B	C	D	E	F
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

3. Complete the truth table for the NOR gate combination



A	B	C	D	E
0	0			
0	1			
1	0			
1	1			

BOOLEAN ALGEBRA

In 1847 George Boole devised a simple method of analysing logic circuits and summarized as below

OR gate; Output= $A+B$

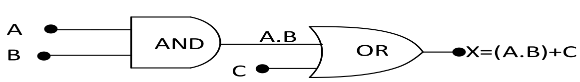
AND gate; Output= $A.B$

NOT gate; Output= $\bar{A}$

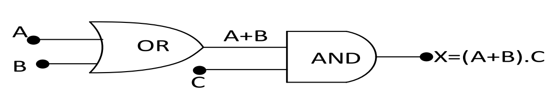
NAND gate; Output= $\overline{A.B}$

NOR gate; Output= $\overline{A+B}$

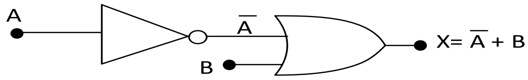
Examples



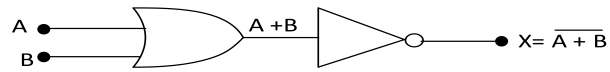
(i)



(ii)

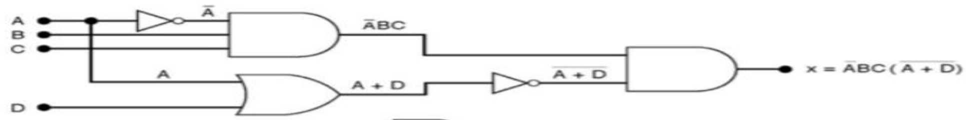


(iii)

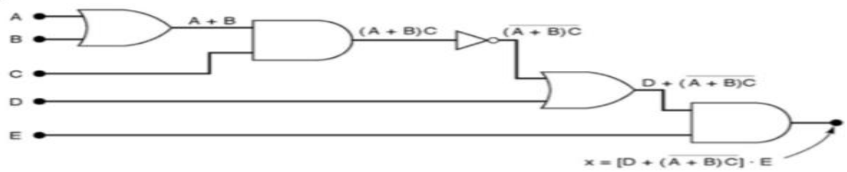


(iv)

(v)

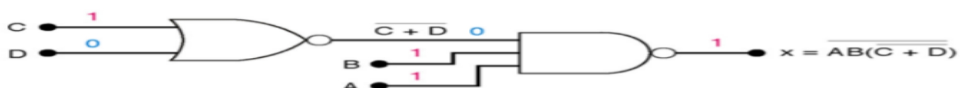


(vi)

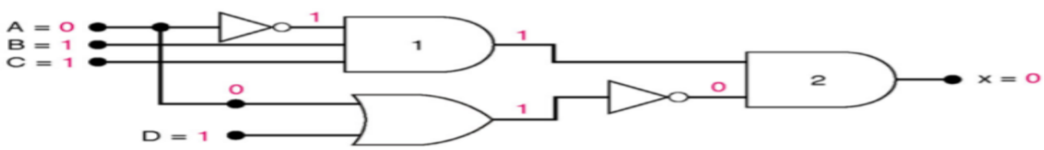


Examples

1.

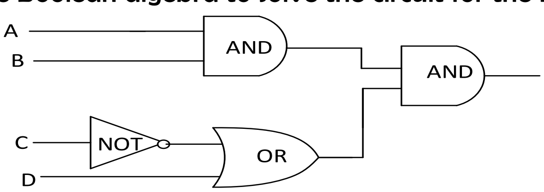


(i)

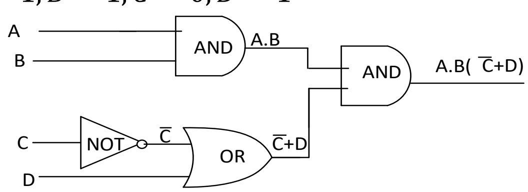


(ii)

2. Use Boolean algebra to solve the circuit for the inputs $A = 1, B = 1, C = 0, D = 1$

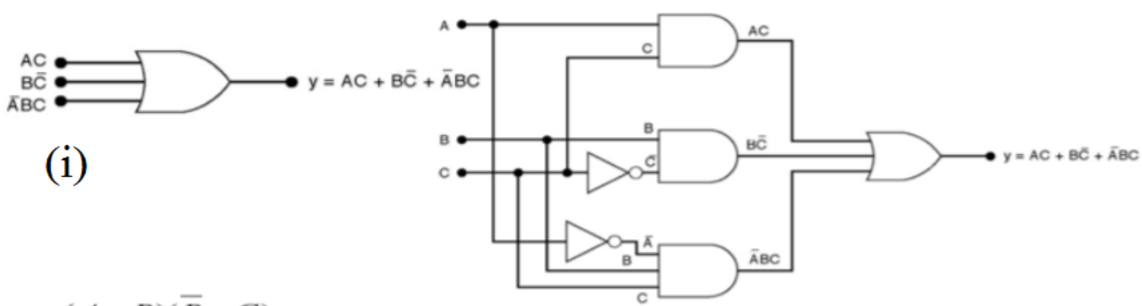


Solution



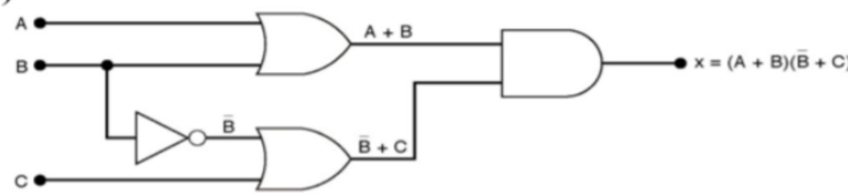
Final ouput; $A.B(\bar{C} + D) = A.B.\bar{C} + A.B.D$
$= 1.1.1 + 1.1.1$
$= 1 + 1 = 1$

Drawing logic circuits from Boolean expression



(i)

$x = (A + B)(\overline{B} + C)$

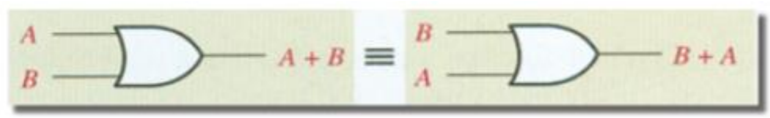


(ii)

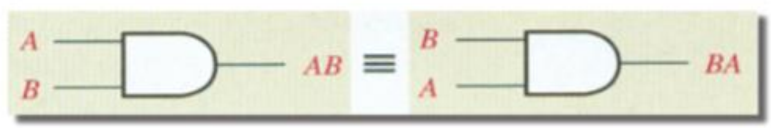
Laws of boolean algebra

- Commutative Laws

A + B = B + A

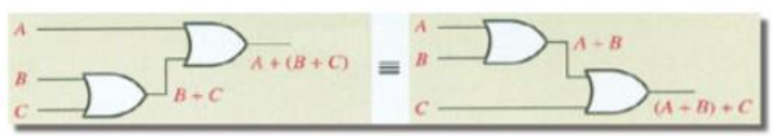


A • B = B • A

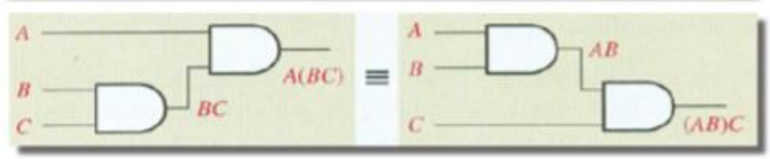


- Associative Laws

A + (B + C) = (A + B) + C

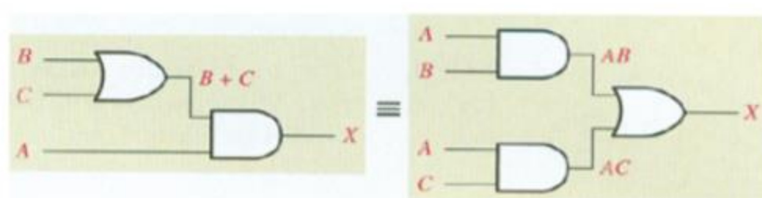


A • (B • C) = (A • B) • C



- Distributive Law

A • (B + C) = A • B + A • C
A (B + C) = A B + A C

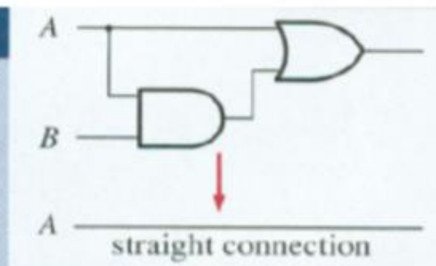


Rules of boolean algebra

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A + A = A$
6. $A + \overline{A} = 1$
7. $A \cdot A = A$
8. $A \cdot \overline{A} = 0$
9. $\overline{\overline{A}} = A$
10. $A + AB = A$
11. $A + \overline{A}B = A + B$
12. $(A + B)(A + C) = A + BC$

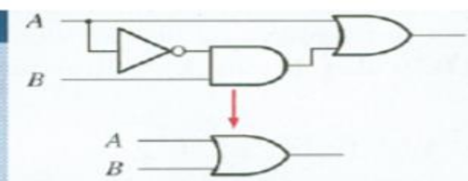
- Rules 1 to 9 are obvious.
- Rule 10: $A + AB = A$

A	B	AB	$A + AB$
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1



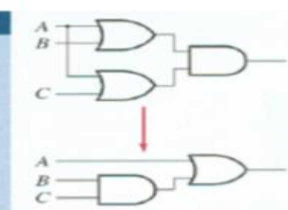
- Rule 11: $A + \overline{A}B = A + B$

A	B	$\overline{A}B$	$A + \overline{A}B$	$A + B$
0	0	0	0	0
0	1	1	1	1
1	0	0	1	1
1	1	0	1	1

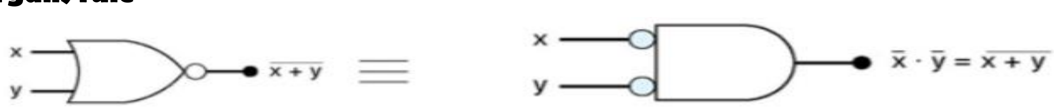


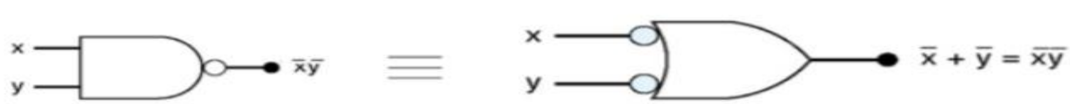
- Rule 12: $(A + B)(A + C) = A + BC$

A	B	C	$A + B$	$A + C$	$(A + B)(A + C)$	BC	$A + BC$
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	1	0	1
1	0	1	1	1	1	0	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1



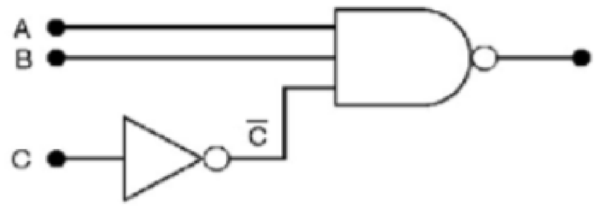
Demorgans rule





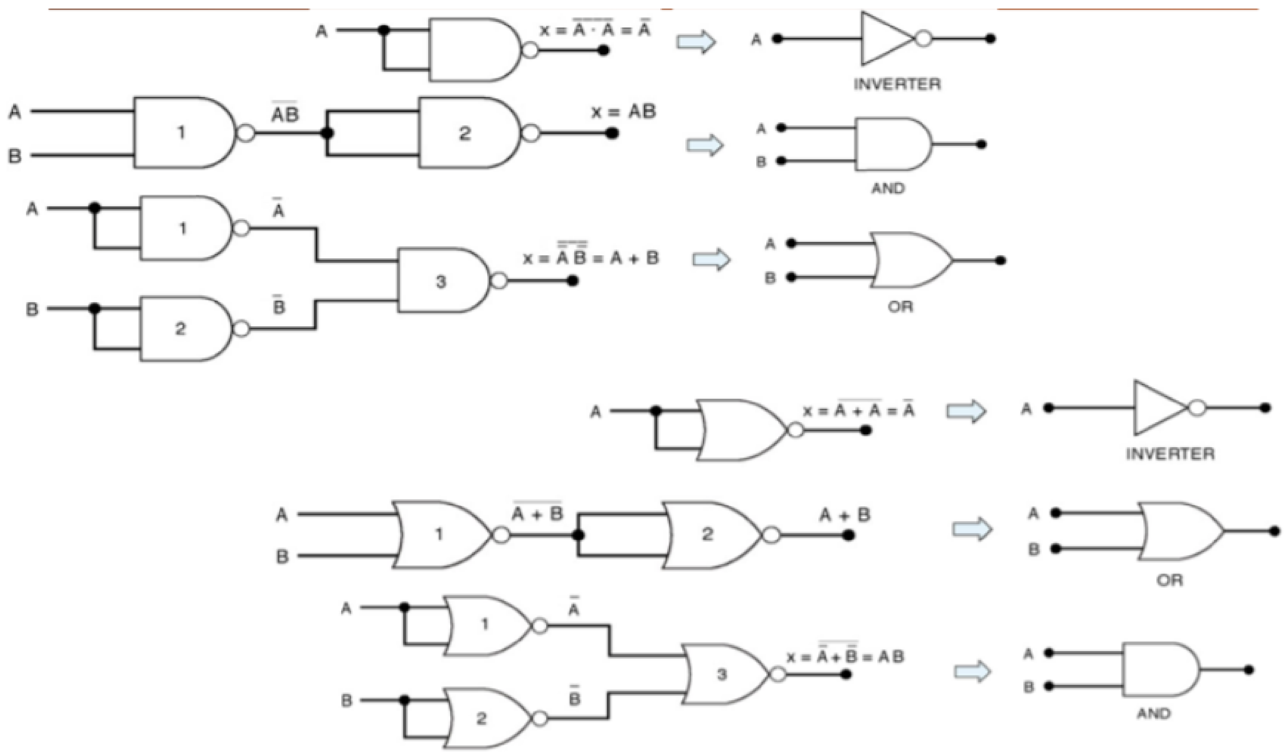
Example

Determine the out put expression for the circuit belowand simplify using Demorgans rule



$z = \overline{A \cdot B \cdot \overline{C}} = \overline{A} + \overline{B} + \overline{\overline{C}} = \overline{A} + \overline{B} + C$

Universality of NAND and NOR gates



UNEB 2020 Q.9

(a) **(i)** What is meant by **a p-n junction** as applied to semiconductors? (01mark)

(ii) Explain the term **doping** as applied to a p-n junction diode. (03marks)

(b) **(i)** Explain, with the aid of a labelled diagram, the I-V charateristics of a junction diode (03marks)

(ii) Describe how full wave rectification can be achieved using a bridge rectifier. (04marks)

(c) The input resistance of a certain n-p-n transistor in the common emitter connection is $3k\Omega$. The small current amplification transfer ratio is 100. The internal resistance of the emitter- base junction is negligible and the load resistor is 6 $k\Omega$. Find the voltage gain. **An 200** (04marks)

UNEB 2019 Q.9

(c) **(i)** What is meant by **thermionic emission?** (01mark)

(ii) Describe how full-wave rectification of a.c can be achieved using four semiconductor diodes (06marks)

UNEB 2012 Q 10

(a) With the aid of a labeled diagram explain full wave rectification [07marks]

(b) i) Sketch the output characteristics of a transistor [02marks]

(ii) Identify on the sketch in e(i) the region over which the transistor can be used as an amplifier. [01mark]

UNEB 2007 Q.8

(a) (i) Describe the structure of a junction transistor [02marks]

(ii) Sketch and describe the collector-current against the collector-emitter voltage characteristics of a junction transistor [03marks]